

Cubic law for fluid flow becomes quintic: An extension of the parallel plate model to natural fracture sets.



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Technical Abstract

The cubic law, the fundamental equation for fluid flow between two parallel plates, has been used for over 100 years. Despite advances in its application to individual geological fractures, it has not yet been extended to fracture sets. We obtain a new relation for the fracture flow rate, Q , using recent results for displacement-to-length scaling, i.e. $D_{max} = \lambda L^{0.5}$, for opening mode fractures (joints, veins and dikes). Evaluation of seven datasets shows that flow rates scale instead by a quintic law. Flow rates in fractures are based on the assumption of laminar flow between two smooth parallel plates and is commonly referred to as "cubic law" because of the proportionality of Q to the cube of the aperture, a , of the plates. It has been shown that, as long as the average opening displacement is used, this parallel plate model accurately predicts flow rates in a single natural fracture. However, in order to gain quantitative information about flow rates in fractured rock, the parallel plate model needs to be extended to fracture sets. Displacement-to-length scaling of opening mode fractures reveal that maximum openings (D_{max}) of fractures relate to their lengths (L) as power law functions with exponents around 0.5. Using a relationship between maximum and average displacement and combining the D_{avg}/L scaling with the parallel plate model reveals that fluid flow rates scale to opening displacements to the fifth power, i.e. a quintic law. This implies that neglecting D/L scaling underestimates fluid flow rates in natural fracture sets.

General Abstract

The cubic law has been used to calculate fluid flow rates for over 100 years. It is based on laminar flow between parallel plates with certain dimensions, i.e. plate length, height and separation distance, and has been applied to single natural fractures. However, it does not take into account that natural fractures occur in fracture sets with a certain scaling relationship between their openings and lengths. Using this scaling relationship reveals that flow scale instead by a quintic law in natural fracture sets.

Objectives

- acquire more D/L measurements that test square root power law dependence
- further develop D/L scaling of Mode I fractures
- combine D/L scaling and fluid flow rates

Fig. 1) Photograph of a pegmatite dike in granite near Emerald Bay, Sierra Nevada, CA, USA. The dike is 30.50m long and has an off-centered maximum displacement of 37cm.

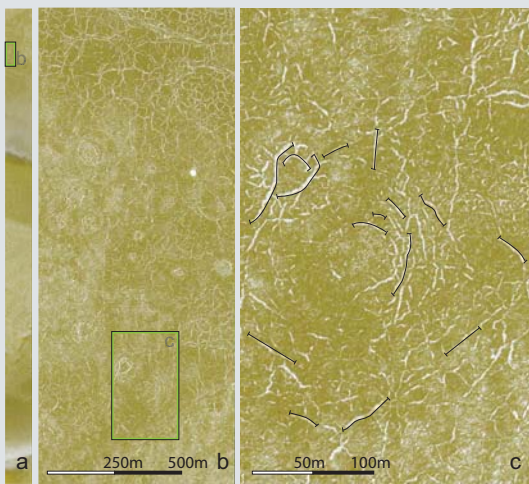


Fig. 2) HiRISE space craft image of Mode I fractures on Mars' surface in the northern plains (67.2°N/47.8°E). In c) the indicated structures were measured.

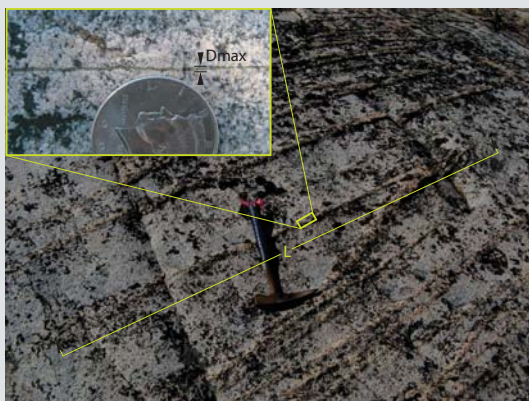


Fig. 3) Set of sub-parallel quartz veins in granite near Emerald Bay, Sierra Nevada, CA, USA.

Glossary:

Stress Intensity factor, K_I , is a parameter that characterizes the magnitude of the near-tip stress concentration that is proportional to the applied load on the material. The minimum value of K_I , required to propagate the crack, is referred to as the critical fracture toughness (K_{Ic}) of the material.

K_{Ic} has units of stress times the root of the length.

Mode I is opening or tensile mode where the crack surfaces move directly apart.

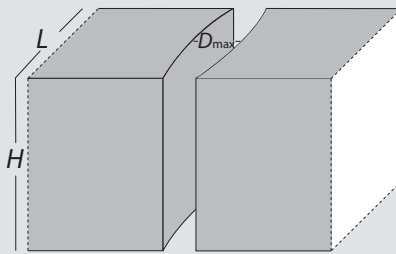
Poisson's ratio (ν), is the ratio of the relative contraction strain, or transverse strain (normal to the applied load), divided by the relative extension strain, or axial strain (in the direction of the applied load).

Young's modulus (E) is a measure of the stiffness of a material. It is also known as the Young modulus, modulus of elasticity, elastic modulus (though the Young's modulus is actually one of several elastic moduli such as the bulk modulus and the shear modulus) or tensile modulus. It is defined as the ratio of stress over strain in the region in which Hooke's Law is obeyed for the material. This can be experimentally determined from the slope of a stress-strain curve created during tests conducted on a sample of the material.

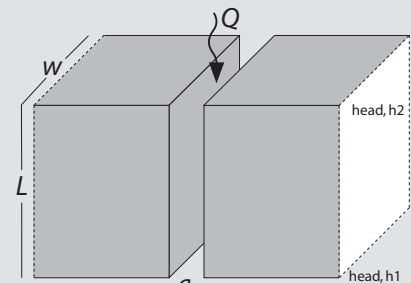
High Resolution Imaging Science Experiment (HiRISE) camera is a camera on board the Mars Reconnaissance Orbiter. The 65 kg, \$40 million instrument was built under the direction of the University of Arizona's Lunar and Planetary Laboratory by Ball Aerospace & Technologies Corp. It consists of a 0.5 meter reflecting telescope, the largest of any deep space mission, which allows it to take pictures with resolutions up to 0.3 m, resolving objects about a meter across. The HiRISE camera is designed to view surface features of Mars in greater detail than has previously been possible.

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Fractures



Parallel Plates



Linkage of parallel plates to fracture sets

$$D_{max} = \lambda L^{0.5} \quad (1)$$

$$Q = \alpha (-a)^3 w \quad (\text{Cubic law}) \quad (3)$$

$$\lambda = \frac{K_{Ic} (1-\nu^2)}{E \sqrt{\pi/8}} \quad (2)$$

$$\alpha = \frac{\rho g}{12\mu} \left(\frac{L}{\Delta h} \right)_f \quad (4)$$

D_{max} Maximum Fracture Aperture
 λ Preexponential Constant
 L Fracture Length
 K_{Ic} Critical Fracture Toughness
 ν Poisson's Ratio
 E Young's Modulus

Q Fluid Flow Rate
 a Average Fracture Aperture
 w Fracture Length
 g Gravitational Constant
 μ Fluid Viscosity
 ρ Fluid Density
 $\left(\frac{L}{\Delta h} \right)_f$ Hydraulic Gradient

$$Q = \frac{\alpha (-a)^3 D_{max}^2}{\lambda^2} \quad (5 [1+3])$$

$$D_{max} = \frac{\pi}{4} a \quad (6)$$

$$Q = \frac{\alpha \left(\frac{\pi}{4} \right)^2 (-a)^5}{\lambda^2} \quad (7 [5+6])$$

(Quintic law)

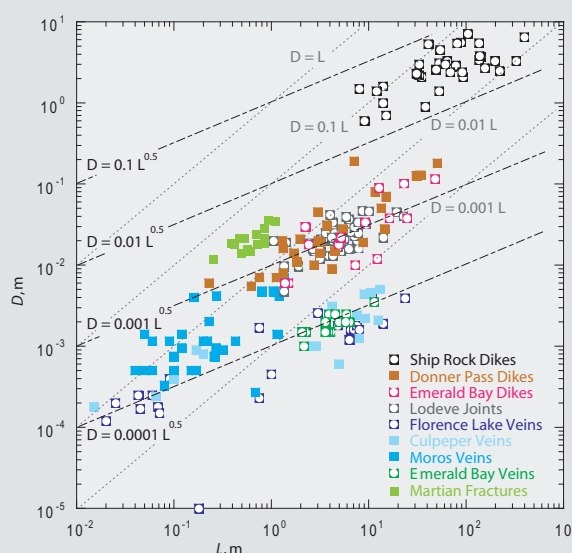


Fig. 4) Displacement-to-length plot of nine Mode I fracture sets with up to 42 individual measurements per set. Fracture lengths scale to fracture apertures as power law functions with exponents around 0.5. For reference, several square root functions indicate power law dependence (black dashed lines) as well as shown linear functions (gray dotted lines) for comparison.

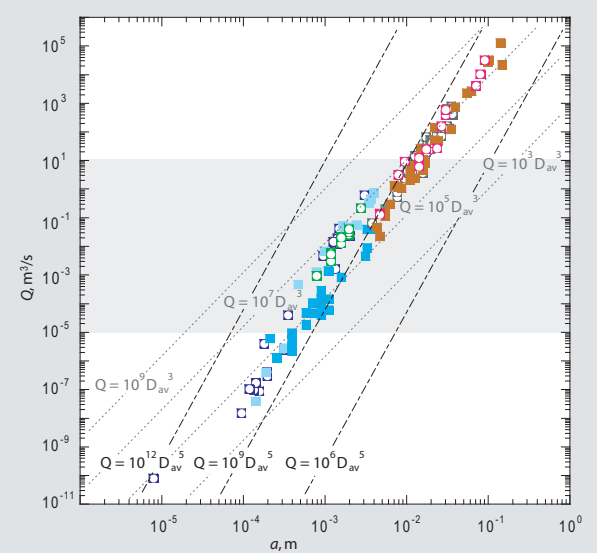


Fig. 5) Flow rates, calculated with the cubic law for water, are scaled against average apertures of the fractures. Note that flow rate-to-aperture relations show a better fit with power law functions with exponents of 5 (black dashed lines) than a cubic dependence (gray dotted lines). For clarity, flow rates in dikes are calculated with water also.

Conclusions

- 4 more D/L datasets support square root power law dependence
- Fluid flow only scales with apertures as a cubic law when there is **no** relation between aperture and length of fracture
- For natural opening mode fracture sets fluid flow scales with apertures to the fifth power

