Cellular Materials: Structure and Properties

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⇨ **DEFINITIONS**

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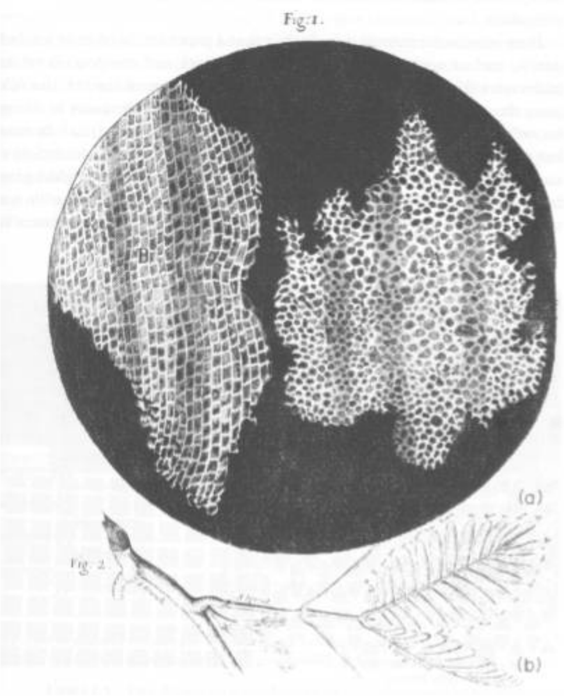
- **CELLULAR MATERIAL**: assembly of *cells* with solid edges and faces, packed together to fill space.
- **CELL** (Robert Hooke, 1660) derives from Latin *cella.*
- **CELLA**: small compartment; an enclosed space.

➥ Robert Hooke, 1660: **CORK** was one of the first materials he examined at his microscope.



Cork ➥ **cellular material**

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⇨ **TYPES** of cellular materials

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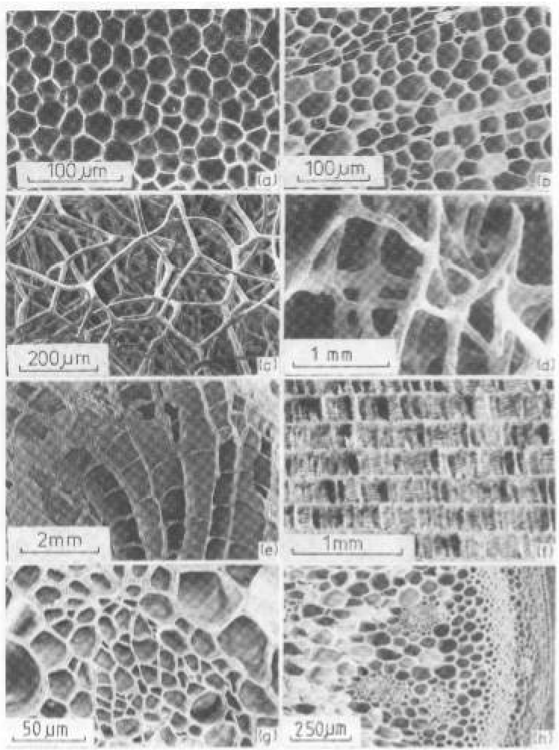
Cellular materials are very common in **Nature**



Cork and Balsa

Sponge and Cancellous Bone

Coral and Cuttlefish Bone

Iris Leaf and Stalk of a Plant

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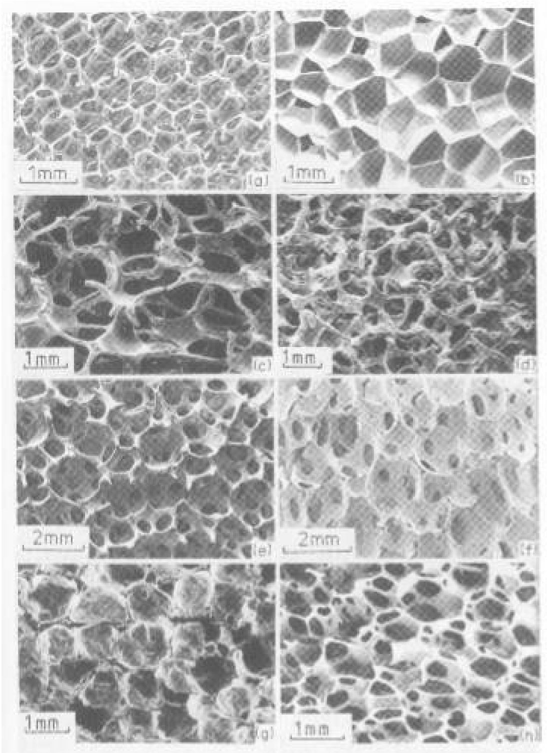
Man makes 3 dimensional (3D) cellular materials

FOAMS



Polymeric materials
Open-cell Polyurethane and Closed-cell Polyethylene

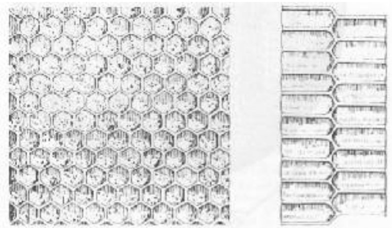
Metallic
Nickel and Copper

Ceramic
Zirconia and Mullite

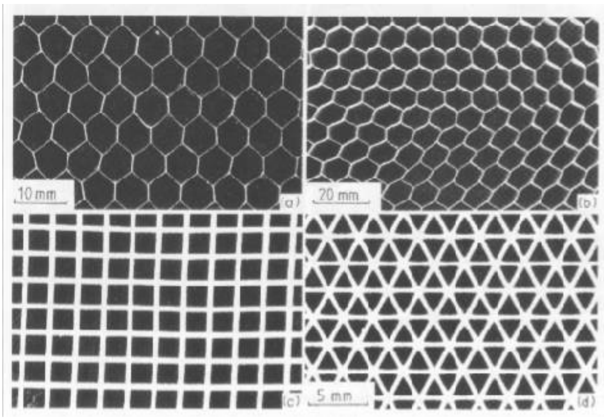
Glass and Polyether

Man makes 2 dimensional (2D) cellular materials

HONEYCOMBS (like a bee)



Honeycomb of a bee

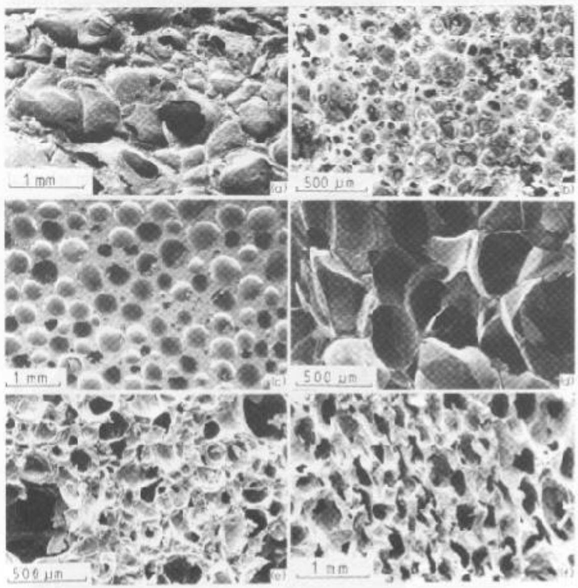


Aluminium and Paper-Phenolic Resin Honeycombs

Ceramic Honeycombs

Many **Foods** are 3D cellular materials

FOAMS



Bread
Meringue

Chocolate
Junk Food Crisp

Malteser
Jaffa Cake

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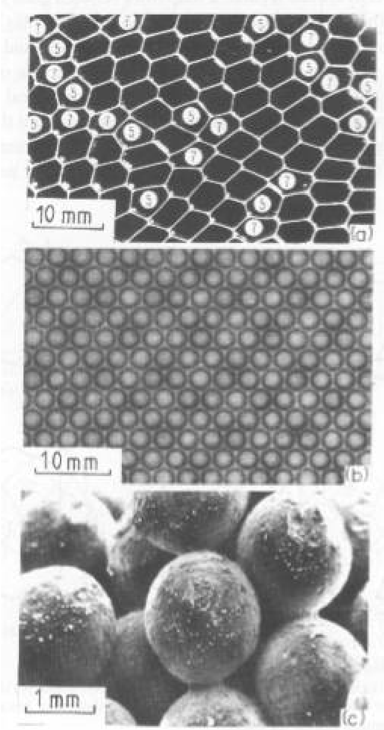
Bubbles can be bonded to give low-density structures

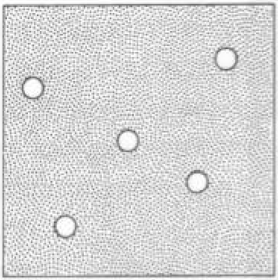


2D Soap Honeycomb

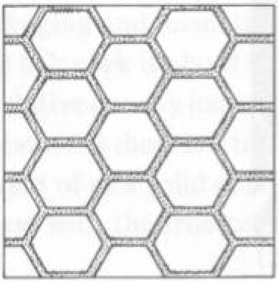
Bubble Raft

Hollow Sintered Aluminium Spheres

What is a cellular material?



Solid with isolated pores; relative density above 0.3



Cellular material; relative density less than 0.3

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⇨ **Types** of cellular materials

- Natural and Man-made
- Two- and Three-dimensional
- Open and Closed Cells

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⇨ **APPLICATIONS** of cellular materials

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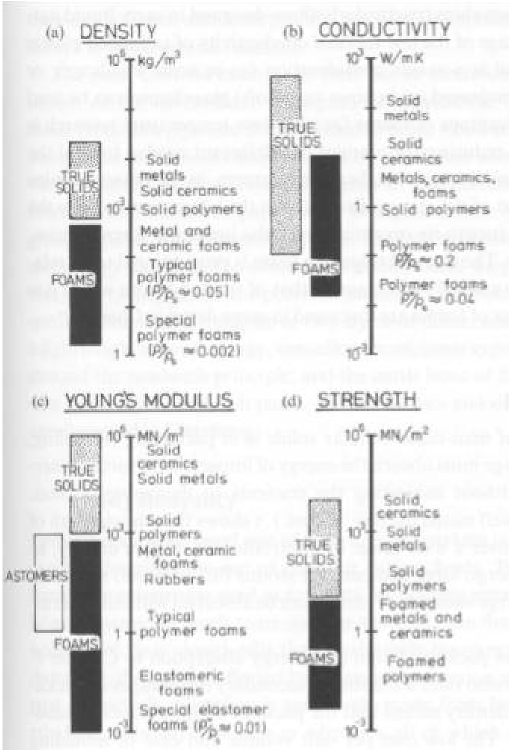
⇨ **Main Applications** of cellular materials

- Thermal insulation
- Packaging
- Structural applications
- Buoyancy

Applications ⇔ **Properties** ⇔ **Structure**

Main Properties of cellular materials



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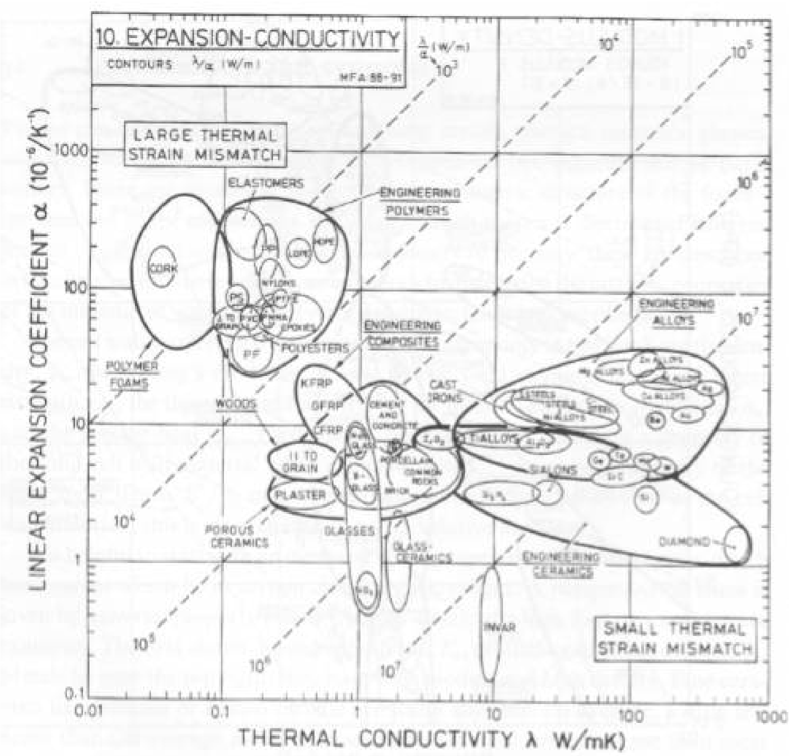
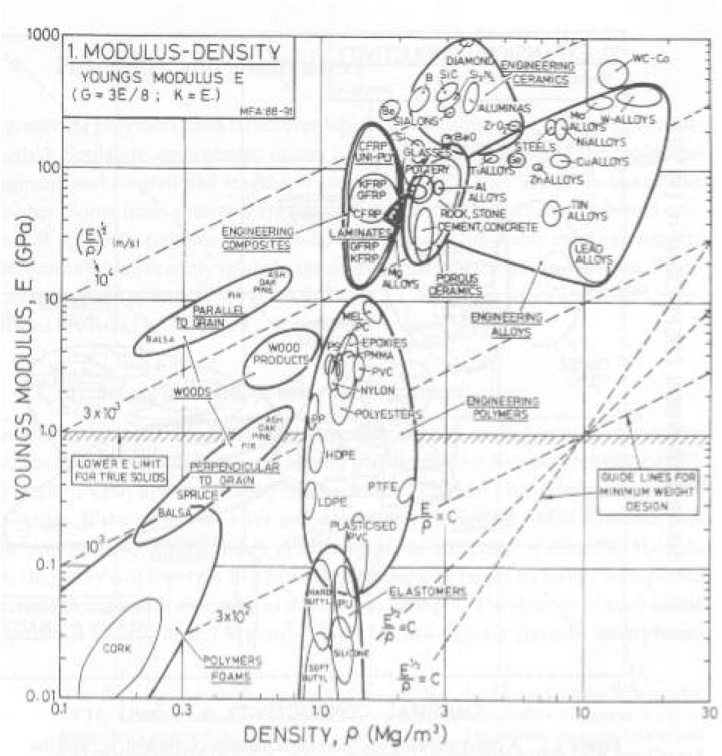
Main Properties of cellular materials



Main Properties of cellular materials



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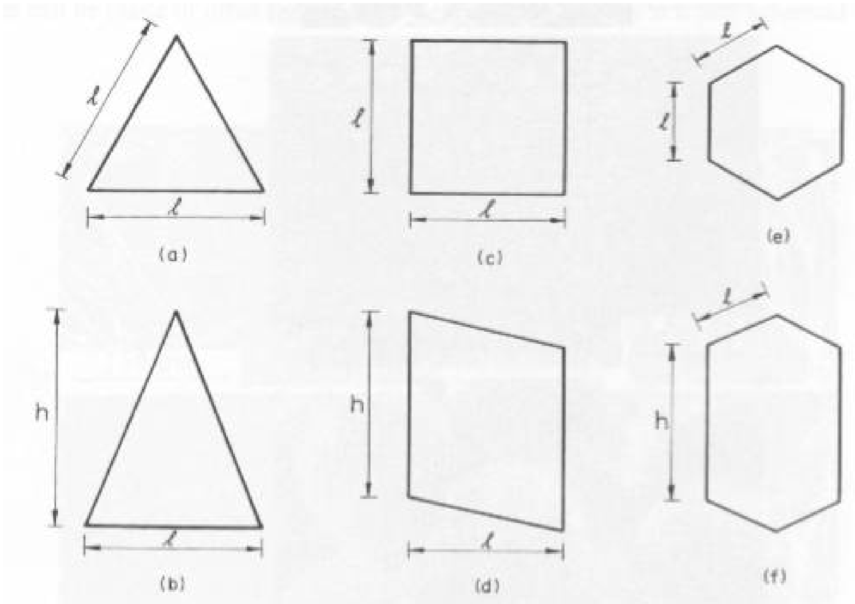
⇨ **STRUCTURE** of cellular materials

Cell **Shape** and **Size**

2D



Triangle (equilateral and isosceles); Square; Paralelogram; Hexagon (regular and irregular)

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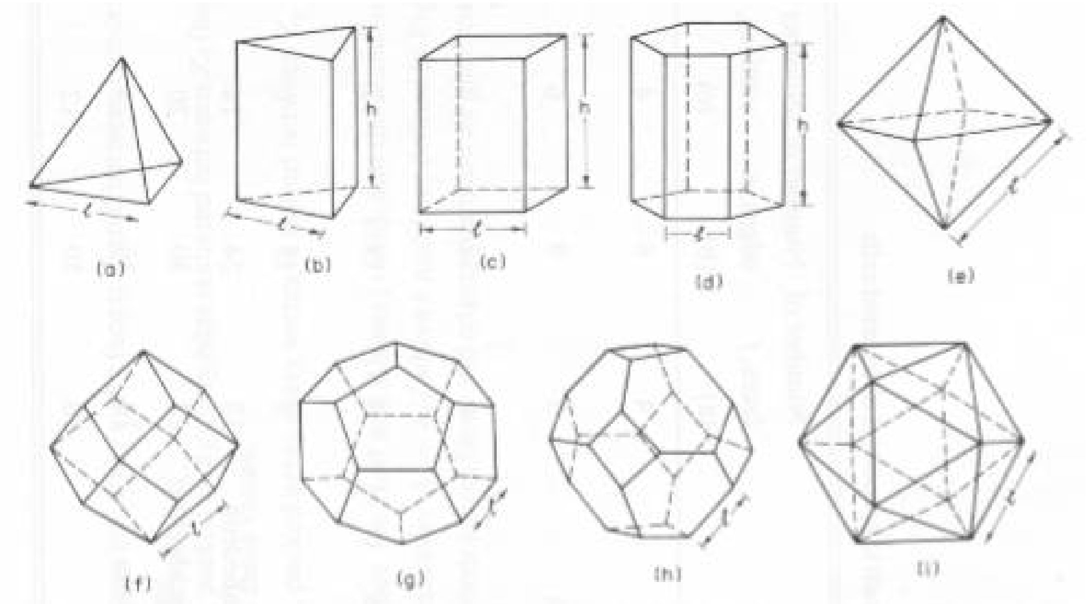
Cell **Shape** and **Size**

3D



Tetrahedron; Prism (triangular, rectangular and hexagonal); Octahedron; Dodecahedron (rhombic and pentagonal); Tetrakaidecahedron; Icosahedron

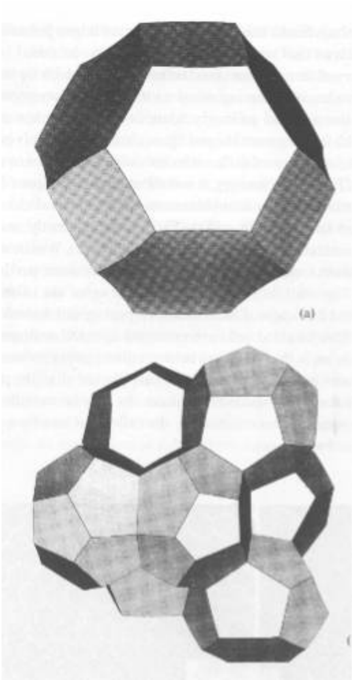
Geometric properties of 3D isolated cells

Cell shape	Number of faces, f (a)	Number of edges, n (b)	Number of vertices, v (c)	Cell volume (d) (e)	Surface area (a) (d) (e)	Edge length (b) (e)
Tetrahedron	4	6	4	$0.118l^3$	$\sqrt{3}l^2$	$6l$
Triangular prism	5	9	6	$\frac{\sqrt{3}}{4} l^3 A_r$	$\frac{\sqrt{3}}{2} l^2(1 + 2\sqrt{3}A_r)$	$6l(1 + A_r/2)$
Square prism	6	12	8	$l^3 A_r$	$2l^2(1 + 2A_r)$	$8l(1 + A_r/2)$
Hexagonal prism	8	18	12	$\frac{3\sqrt{3}}{2} l^3 A_r$	$3\sqrt{3}l^2(1 + 2A_r/\sqrt{3})$	$12l(1 + A_r/2)$
Octahedron	8	12	6	$0.471l^3$	$3.46l^2$	$12l$
Rhombic Dodecahedron	12	24	14	$2.79l^3$	$10.58l^2$	$24l$
Pentagonal Dodecahedron	12	30	20	$7.663l^3$	$20.646l^2$	$30l$
Tetrakaidecahedron	14	36	24	$11.31l^3$	$26.80l^2$	$36l$
Icosahedron	20	30	12	$2.182l^3$	$8.660l^2$	$30l$

$$A_r = h/l$$

Cell **Shape** and **Size**

3D



Kelvin tetrakaidecahedral cell

Weaire&Phelan's unit cell (six 14-faced polyhedra + two 12-faced polyhedra)

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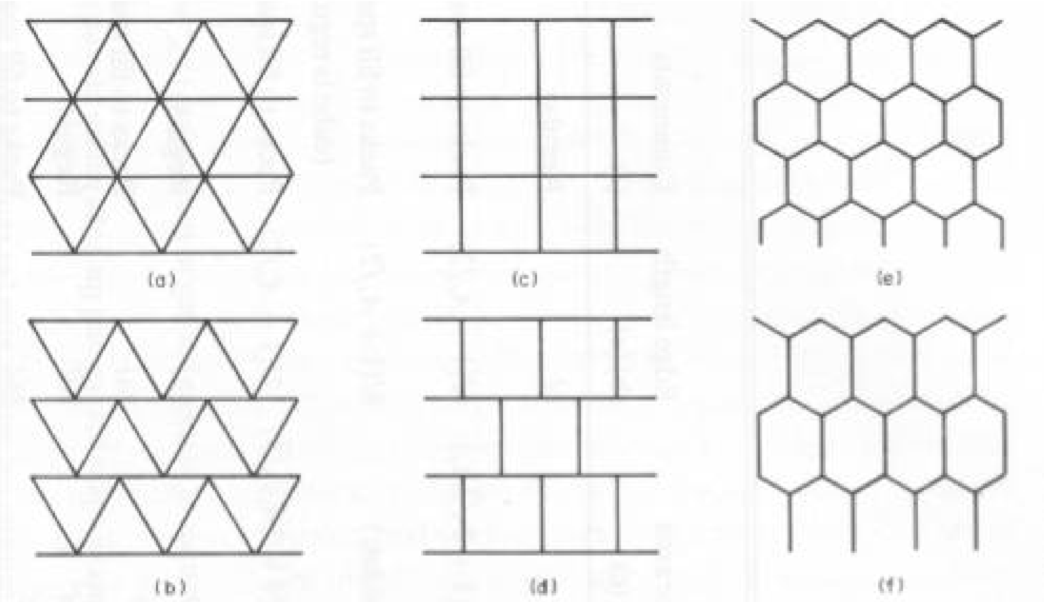
Cell **Topology**

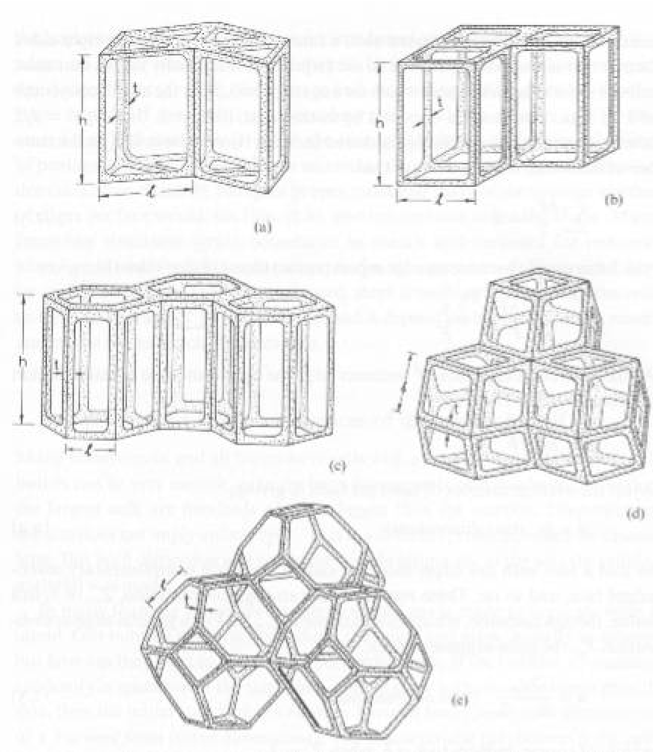
2D



Different edge connectivity (number of edges in each vertice)

Cell **Topology**

3D



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- **Euler's law**: cells (C), faces (F), edges (E) and vertices (V)

2D: $F - E + V = 1$ ($C = F$)

3D: $-C + F - E + V = 1$

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2D

$Z_e = 3$ $E/V = 3/2$ (each edge is shared between 2 vertices)

$F_n =$ number of faces with n sides $\sum \frac{nF_n}{2} = E$

From Euler's law $\left(6 - \frac{\sum nF_n}{F}\right) = \frac{6}{F}$

If F is large $\frac{\sum nF_n}{F} = \bar{n} = 6$

For any Z_e $\bar{n} = \frac{2Z_e}{Z_e - 2}$

3D

An isolated cell (C=1)

$Z_e = 3$

$$\bar{n} = 6\left(1 - \frac{2}{f}\right)$$

$f = 12$ $\bar{n} = 5$

$f = 14$ $\bar{n} = 5.14$

For any Z_e and Z_f

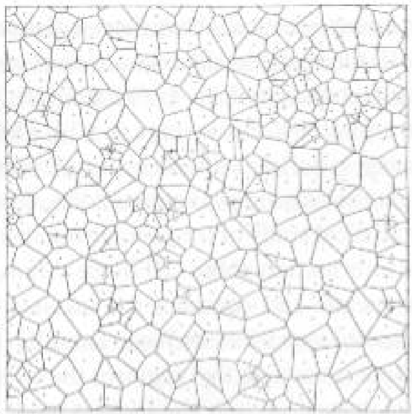
$$\bar{n} = \frac{Z_e Z_f}{Z_e - 2}\left(1 - \frac{2}{f}\right)$$

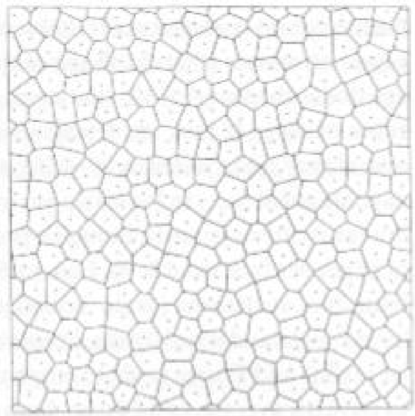
⇨ Dispersion of cell size

Competitive growth

Voronoi honeycomb (2D) or foam (3D)

Bubbles nucleate randomly in space at the same time and all grow with the same linear growth rate ⇨ characteristic cell centred on the point of nucleation and contains all points which are closer to this nucleation point than to any other.





Random Voronoi honeycomb and Voronoi honeycomb for a set of points initially random, from which all points closer than a critical spacing were removed

Surface tension (T)

Minimizes surface area at constant cell volume; cell edges in a honeycomb and cell faces in a foam meet at 120º; faces have a curvature which is related to the pressure difference Δp between the pair of cells which meet at the faces.

$$\Delta p = \frac{T}{r} \qquad (2D)$$

$$\Delta p = T\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \qquad (3D)$$

Coarsening of cells

Important in soap foams and during foaming of polymers.
Gas or fluid in one cell diffuses through its walls into the surrounding cells.

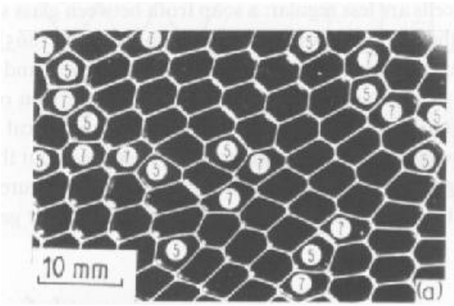
$$\frac{\mathrm{d}A}{\mathrm{d}t} = C_1(n - 6) \qquad (2D)$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = C_2(f - \bar{f}) \qquad (3D)$$

$$\frac{A(n)}{A(\bar{n})} = \frac{n - n_0}{\bar{n} - n_0} \qquad (2D)$$

$$\frac{V(f)}{V(\bar{f})} = \frac{f - f_0}{\bar{f} - f_0} \qquad (3D)$$

2D



$Z_e = 3 \quad \Rightarrow \quad \bar{n} = 6$ $\qquad\qquad n = 7 \quad \Rightarrow \quad n = 5$

A cell with more sides than average has neighbours which, taken together, have less than the average number.

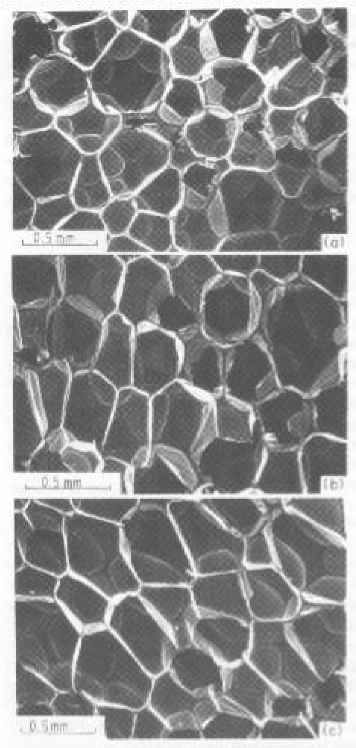
$$\overline{m} = 5 + \frac{6}{n}$$

3D

$\bar{f} = 14$ $\qquad\qquad f = 16 \quad \Rightarrow \quad f = 12$

$$\bar{g} = 13 + \frac{14}{f}$$

Anisotropy



Three orthogonal sections of an anisotropic polyurethane foam

Relative density (ρ/ρ_S)

Honeycombs	
Equilateral triangles ($Z_e = 6, n = 3$ or $Z_e = 4, n = 4$)†	$\frac{\rho^*}{\rho_s} = 2\sqrt{3}\frac{t}{l}\left(1 - \frac{\sqrt{3}}{2}\frac{t}{l}\right)$
Squares ($Z_e = 4, n = 4$ or $Z_e = 3, n = 6$)†	$\frac{\rho^*}{\rho_s} = 2\frac{t}{l}\left(1 - \frac{1}{2}\frac{t}{l}\right)$
Regular hexagons ($Z_e = 3, n = 6$)	$\frac{\rho^*}{\rho_s} = \frac{2}{\sqrt{3}}\frac{t}{l}\left(1 - \frac{1}{2\sqrt{3}}\frac{t}{l}\right)$
Three dimensions: open cells (aspect ratio $A_r = h/l$)	
Triangular prisms ($Z_e = 8, Z_f = 4.5, n = 3.6, f = 5$)	$\frac{\rho^*}{\rho_s} = \frac{2}{\sqrt{3}}\frac{t^2}{l^2}\left\{1 + \frac{3}{A_r}\right\}$
Square prisms ($Z_e = 6, Z_f = 4, n = 4, f = 6$)	$\frac{\rho^*}{\rho_s} = \frac{t^2}{l^2}\left\{1 + \frac{2}{A_r}\right\}$
Hexagonal prisms ($Z_e = 5, Z_f = 3.6, n = 4.5, f = 8$)	$\frac{\rho^*}{\rho_s} = \frac{4}{3\sqrt{3}}\frac{t^2}{l^2}\left\{1 + \frac{3}{2A_r}\right\}$
Rhombic dodecahedra ($Z_e = 5.33, Z_f = 3, \bar{n} = 4, \bar{f} = 12$)	$\frac{\rho^*}{\rho_s} = 2.87\frac{t^2}{l^2}$
Tetrakaidecahedra ($Z_e = 4, Z_f = 3, \bar{n} = 5.14, \bar{f} = 14$)	$\frac{\rho^*}{\rho_s} = 1.06\frac{t^2}{l^2}$
Three dimensions: closed cells (aspect ratio $A_r = h/l$)	
Triangular prisms ($Z_e = 8, Z_f = 4.5, n = 3.6, f = 5$)	$\frac{\rho^*}{\rho_s} = 2\sqrt{3}\frac{t}{l}\left\{1 + \frac{1}{2\sqrt{3}A_r}\right\}$
Square prisms ($Z_e = 6, Z_f = 4, \bar{n} = 4, \bar{f} = 6$)	$\frac{\rho^*}{\rho_s} = 2\frac{t}{l}\left\{1 + \frac{1}{2A_r}\right\}$
Hexagonal prisms ($Z_e = 5, Z_f = 3.6, \bar{n} = 4.5, \bar{f} = 8$)	$\frac{\rho^*}{\rho_s} = \frac{2}{\sqrt{3}}\frac{t}{l}\left\{1 + \frac{\sqrt{3}}{2A_r}\right\}$
Rhombic dodecahedra ($Z_e = 5.33, Z_f = 3, \bar{n} = 4, \bar{f} = 12$)	$\frac{\rho^*}{\rho_s} = 1.90\frac{t}{l}$
Tetrakaidecahedra ($Z_e = 4, Z_f = 3, \bar{n} = 5.14, \bar{f} = 14$)	$\frac{\rho^*}{\rho_s} = 1.18\frac{t}{l}$

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Characterization chart for honeycombs

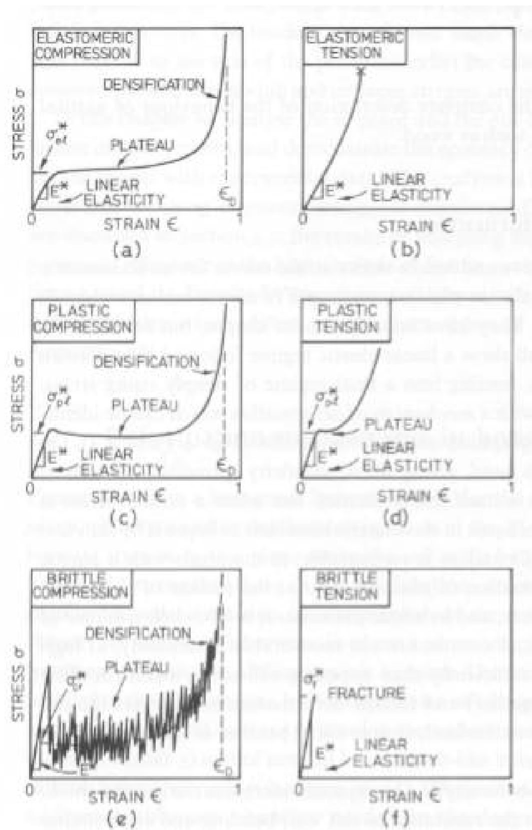
Material	Alumina ceramic (Fig. 2.3(c))
Density, ρ^* (kg/m^3)	1400
Edge connectivity, Z_e	4
Mean edges/cell, $\bar{n}$	4
Cell shape (and angles)	Square
Symmetry of structure	Square (mm)
Largest principal cell dimension, L_1 (mm)	2.42
Smallest principal cell dimension, L_2 (mm)	2.42
Shape anisotropy ratio, $R = L_1/L_2$	1
Standard deviation of cell size	0
Cell wall thickness, t (mm)	0.48
Relative density, ρ^*/ρ_s	0.36
Other specific features (periodic variations in density, cell size, etc.)	Highly regular

Characterization chart for foams

Material	Rigid polyurethane (Fig. 2.19)
Density, ρ^* (kg/m^3)	32
Open or closed cells	Closed
Edge connectivity, Z_e	4
Face connectivity, Z_f	3
Mean edges/face, $\bar{n}^*$	—
Mean faces/cell, $\bar{f}^*$	—
Cell shape*	—
Symmetry of structure	Axisymmetric
Cell edge thickness, t_e (μm)	30
Cell face thickness, t_f (μm)	3
Fraction of material in cell edges, ϕ	0.70
Largest principal cell dimension, L_1 (mm)	0.53
Smallest principal cell dimension, L_3 (mm)	0.44
Intermediate principal cell dimension, L_2 (mm)	0.53
Shape anisotropy ratios, $R_{12} = L_1/L_2$ and $R_{13} = L_1/L_3$	$R_{12} = 1.0$; $R_{13} = 1.2$
Standard deviation of cell size (mm)	0.075
Other specific features (periodic variations in density, cell size, etc.)	None

⇨ **MECHANICS** of honeycombs

Compression and tension stress-strain curves



Elastomeric Honeycomb

Elastic-plastic Honeycomb

Elastic-brittle Honeycomb

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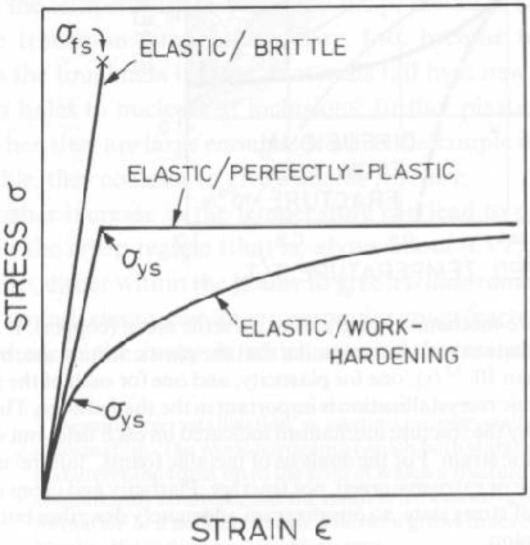
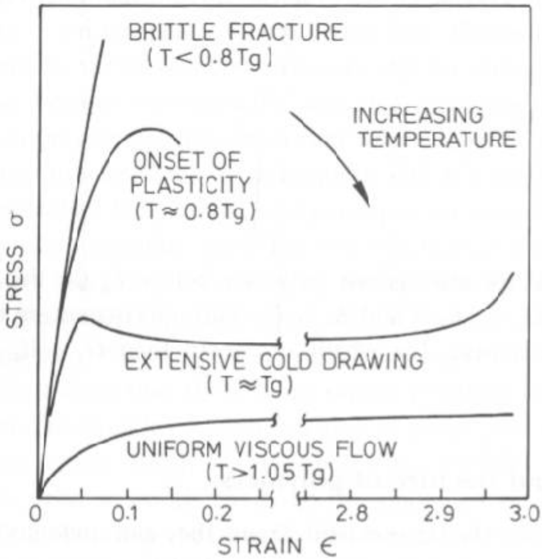
Schematic stress-strain curves of compact materials

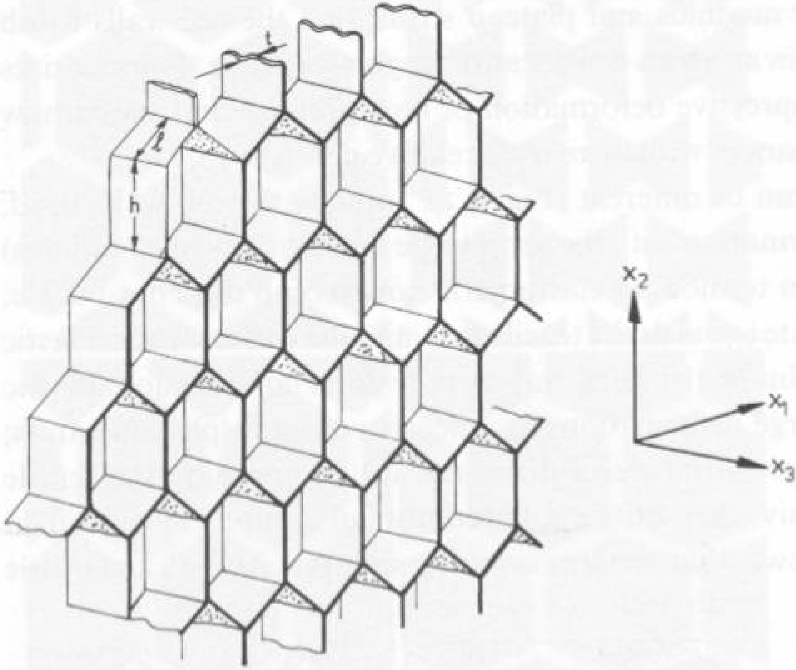




Metals are elastic-plastic
Ceramics are elastic-brittle

Polymers

↳ **Model** of a honeycomb with hexagonal cells



In-plane properties are those relating to loads applied in the X1-X2 plane; responses to loads applied in the X3 direction are referred to as the out-of-plane properties

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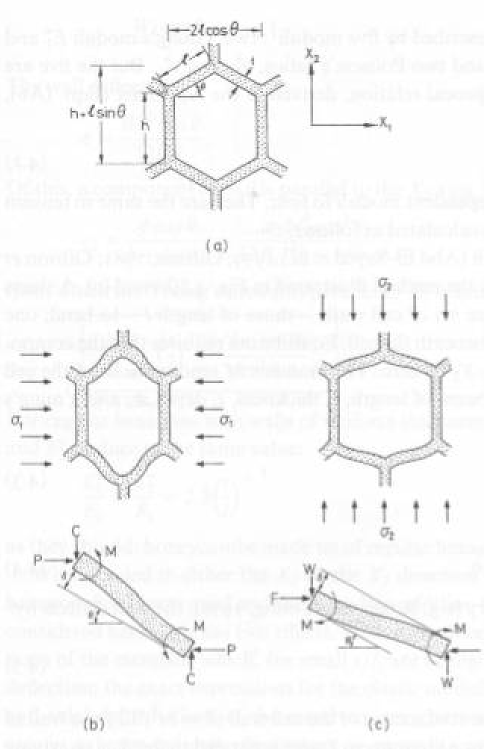
⇨ In-plane deformation

↳ Elastic Bending



$$P = \sigma_1 (h + \ell \sin\theta) b \qquad W = \sigma_2 \ell b \cos\theta$$

$$M = \frac{P \ell \sin\theta}{2} \qquad M = \frac{W \ell \cos\theta}{2}$$

$$\delta = \frac{P \ell^3 \sin\theta}{12 E_S I} \qquad \delta = \frac{W \ell^3 \cos\theta}{12 E_S I} \qquad I = \frac{b t^3}{12}$$

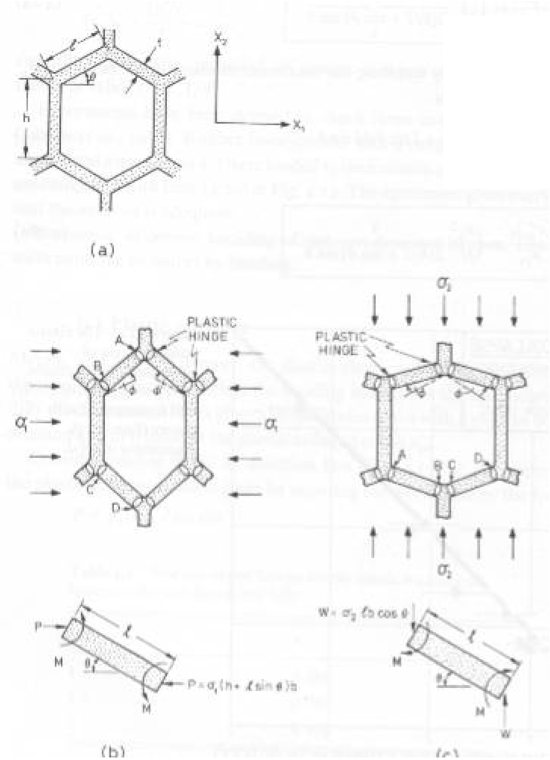
$$\varepsilon_1 = \frac{\delta \sin\theta}{\ell \cos\theta} \qquad \varepsilon_2 = \frac{\delta \cos\theta}{h + \ell \sin\theta}$$

$$\frac{E_1}{E_S} = \left(\frac{t}{\ell}\right)^3 \frac{\cos\theta}{\left(\frac{h}{\ell} + \sin\theta\right)\sin^2\theta} \qquad \frac{E_2}{E_S} = \left(\frac{t}{\ell}\right)^3 \frac{\left(\frac{h}{\ell} + \sin\theta\right)}{\cos^3\theta}$$

$$\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1} = \frac{\cos^2\theta}{\left(\frac{h}{\ell} + \sin\theta\right)\sin\theta} = \frac{1}{\nu_{21}} \qquad E_1 \nu_{21} = E_2 \nu_{12}$$

⇨ **In-plane** deformation

↳ Plastic Collapse



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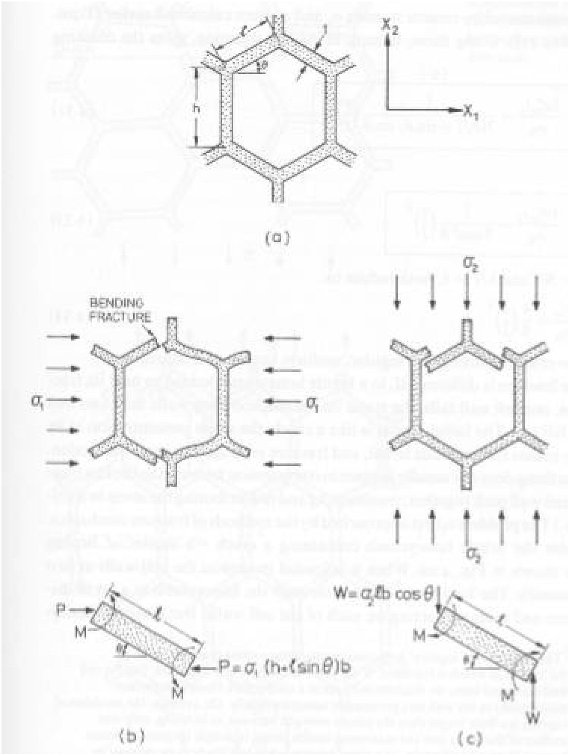
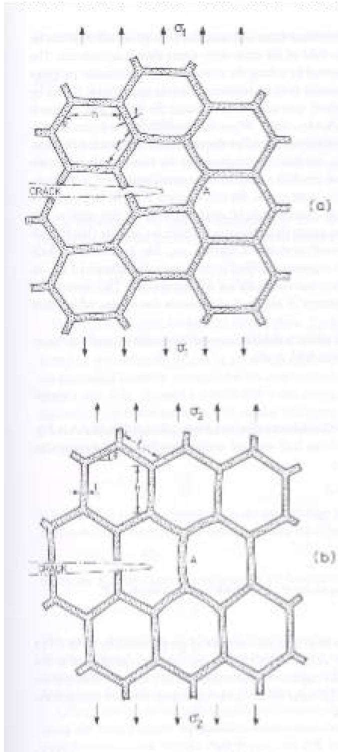
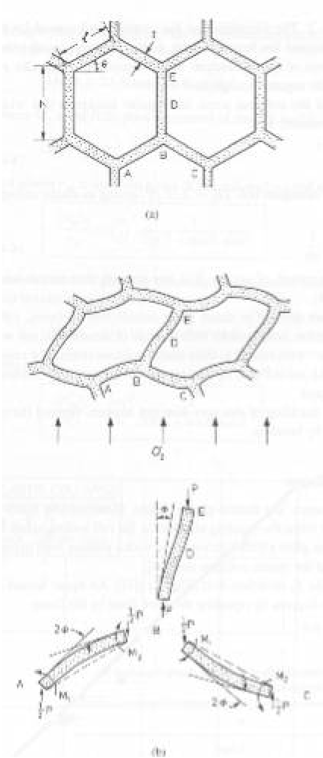
- ⇨ **In-plane** deformation
- ⇨ Brittle Fracture and Crack Propagation

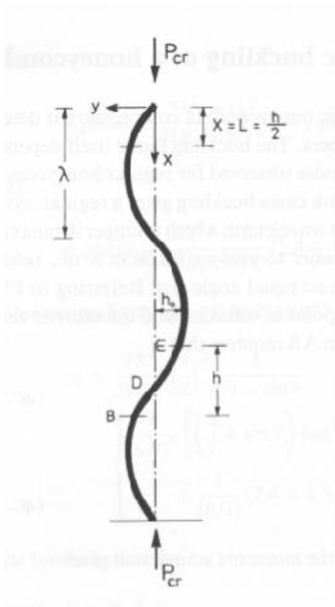




⇨ In-plane deformation

Buckling

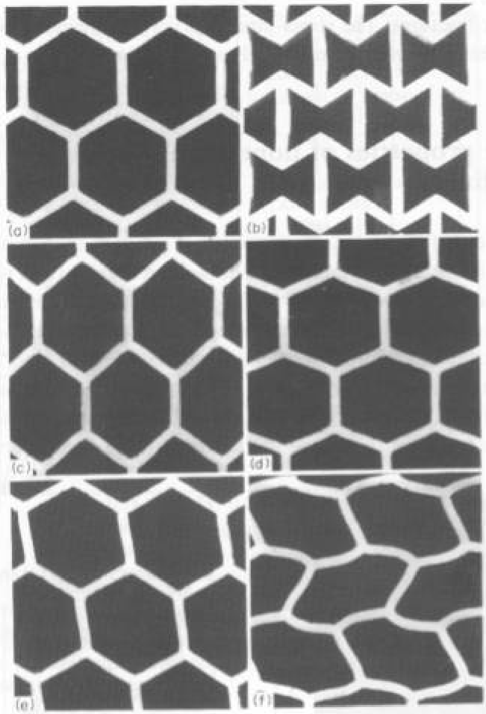




$$\text{Euler buckling load} = P_{crit} = \frac{n^2 \pi^2 E_S I}{h^2}$$

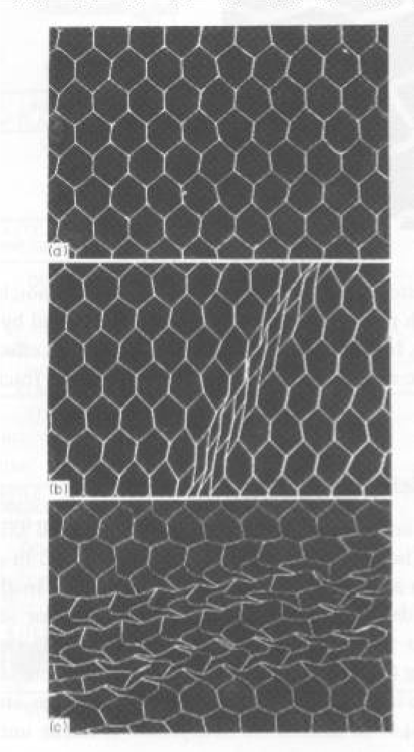
➯ **In-plane** deformation

➯ Mechanisms of deformation in rubber honeycomb



⇨ **In-plane** deformation

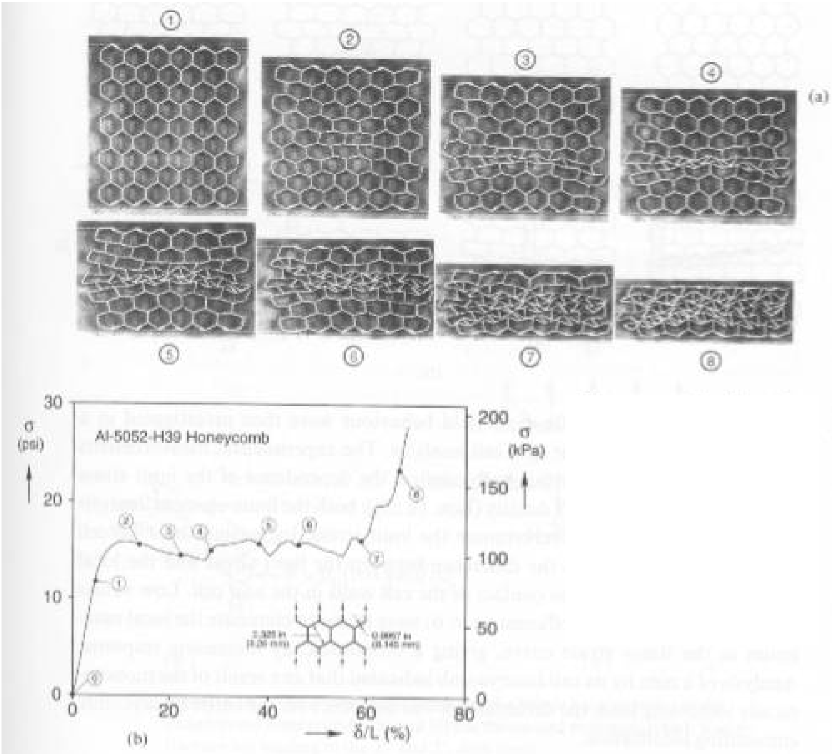
⇨ Mechanisms of deformation in aluminium honeycomb



Localized deformation

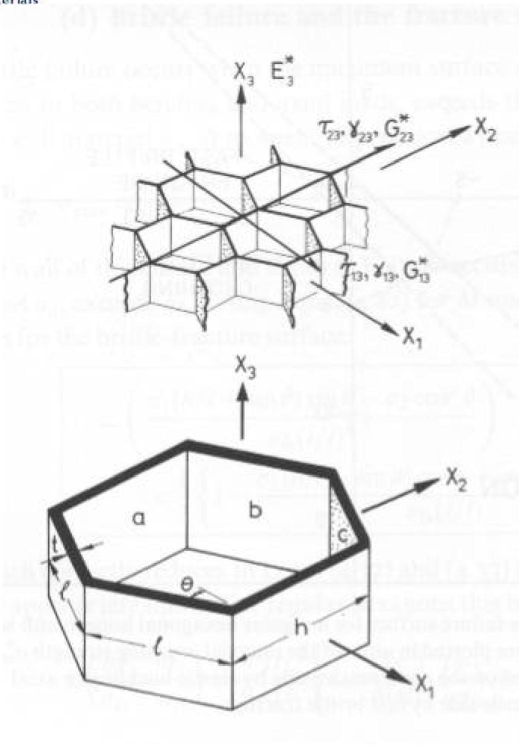
⇨ **In-plane** deformation

Crushing of aluminium honeycomb



⇨ **Out of-plane** deformation

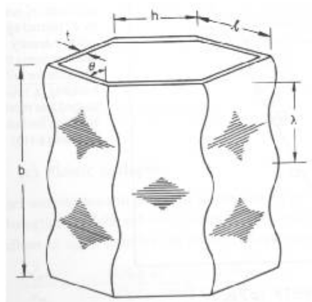
↳ Compression and Buckling



$$\frac{E_3}{E_S} = \left[\frac{\frac{\eta}{\ell} + 2}{2\left(\frac{\eta}{\ell} + \sin\theta\right)\cos\theta} \right] \frac{\tau}{\ell} = \frac{\rho}{\rho_S}$$

$$\nu_{31} = \nu_{32} = \nu_S$$

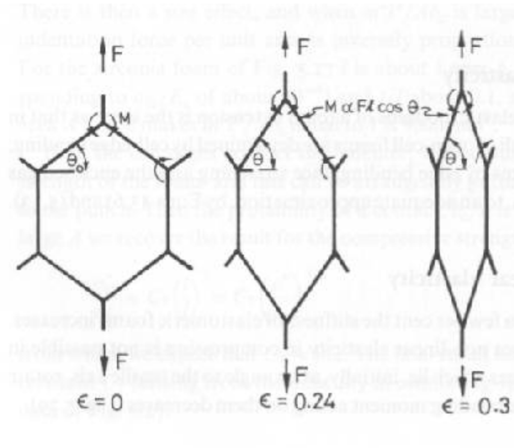
$$E_1\,\nu_{31} = E_3\,\nu_{13} \approx 0 \qquad E_2\,\nu_{32} = E_3\,\nu_{23} \approx 0$$



Buckling

Tension

Alignment of cell edges



No buckling

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⇨ **MECHANICS** of foams

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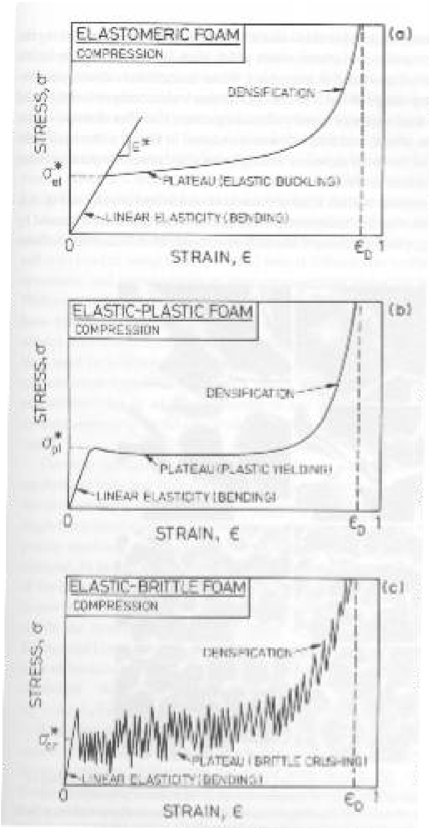
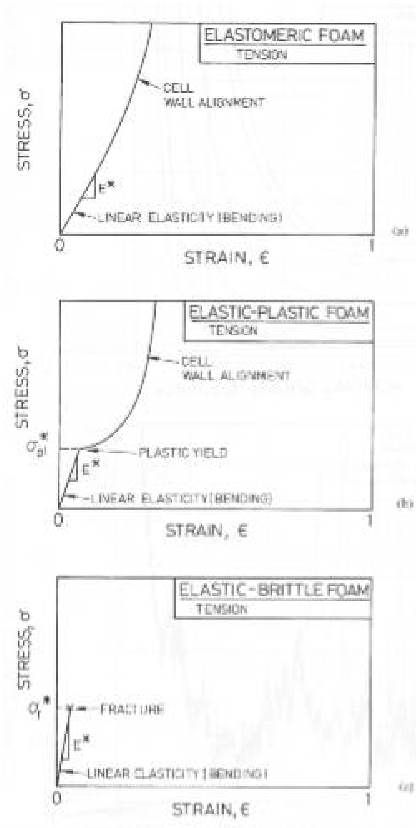
Compression and tension stress-strain curves



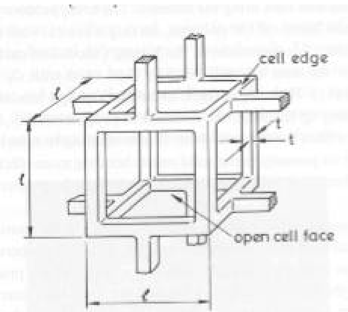


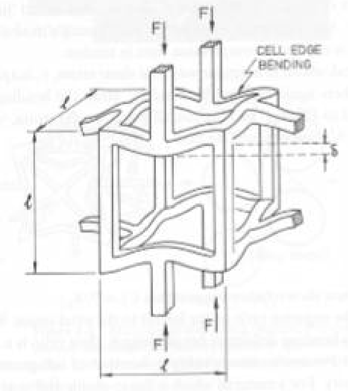
Elastomeric Honeycomb

Elastic-plastic Honeycomb

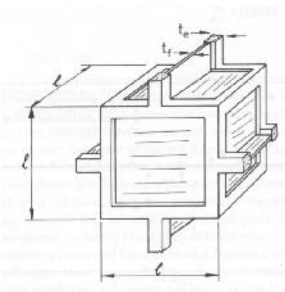
Elastic-brittle Honeycomb

Models of foams with open/closed cubic cells





Bending



Solid concentrates in edges

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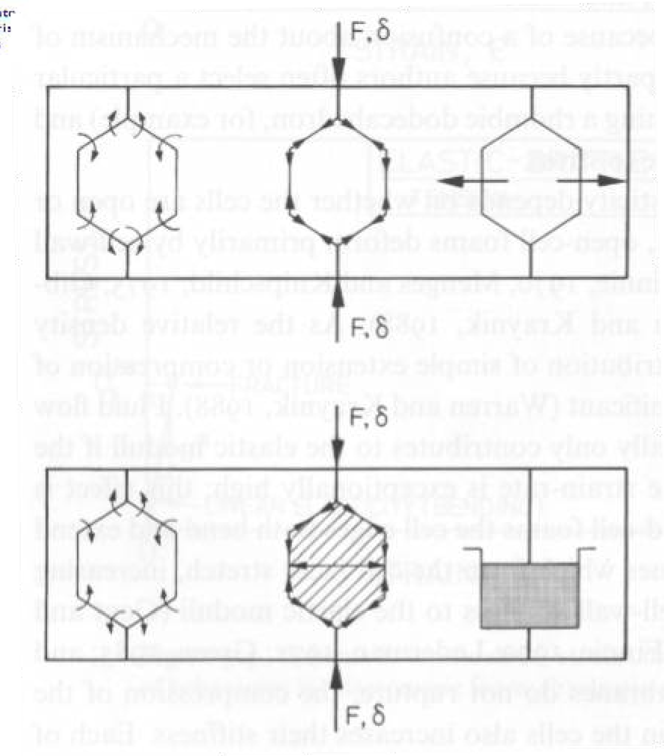
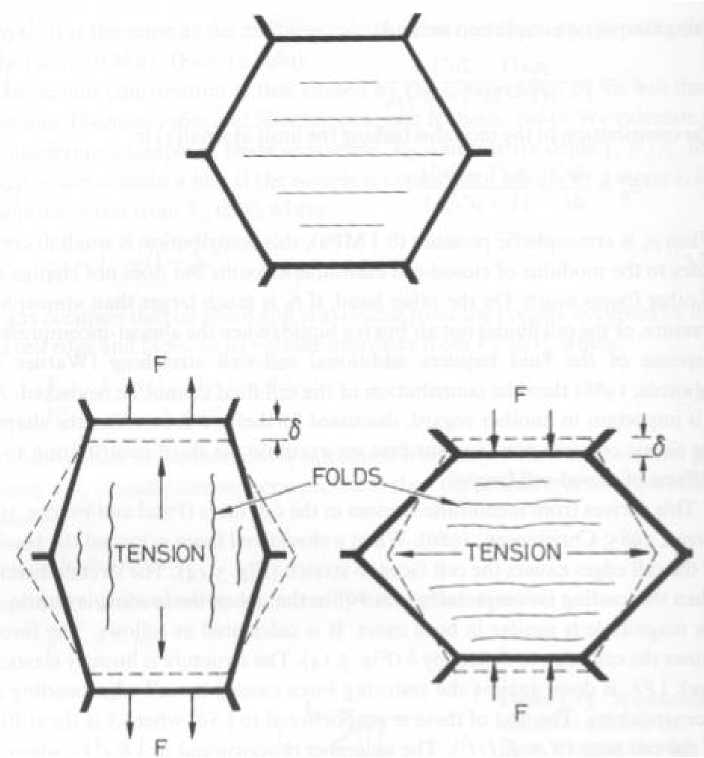
Mechanisms of deformation in foams



Open-cell Foams:
wall bending and axial deformation + fluid flow

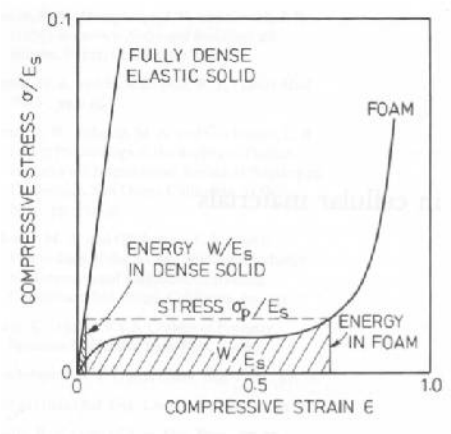
Closed-cell Foams:
wall bending and axial deformation + edge contraction and membrane stretching + enclosed gas pressure

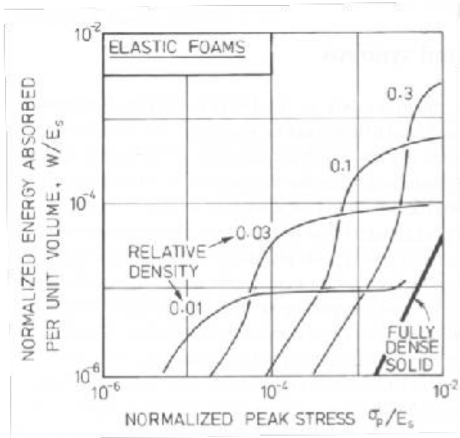
Membrane effects



Bending of cell edges causes cell faces to stretch

Energy absorption in foams



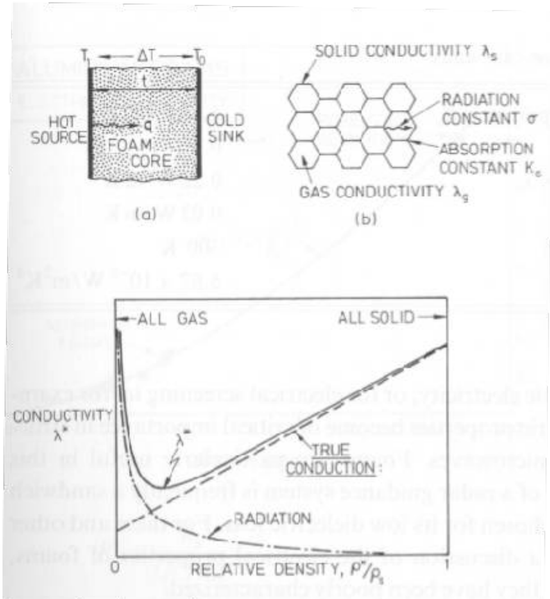


Thermal conductivity in foams

$$\theta = -\lambda \nabla T$$

$$\alpha = \frac{\lambda}{\rho X_p}$$

$$\lambda = \lambda_S + \lambda_G + \lambda_c + \lambda_r$$



Convection is suppressed

***Optimum foam density* for minimum conduction**

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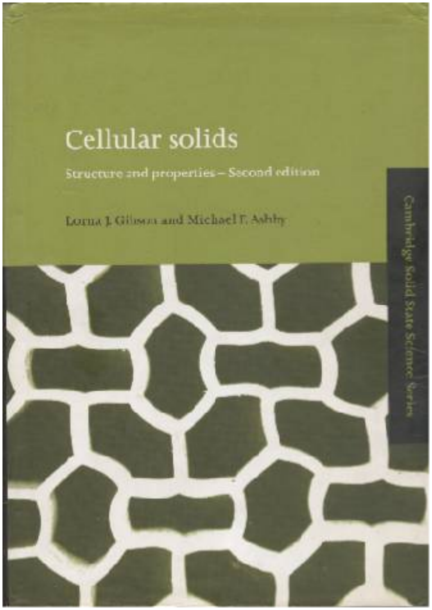

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Weaire, D. and Hutzler, S. (1999). *The Physics of Foams*. Clarendon Press, Oxford