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Rate controlling processes in the creep of polar ice, influence of grain boundary migration associated with recrystallization

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Abstract

Information on the deformation modes and recrystallization processes in polar ice sheets was obtained thanks to the analysis of the ice structure along two deep ice cores from Greenland and Antarctica. It is shown that the deformation of polar ice at low stresses is produced by intracrystalline slip accommodated by grain boundary migration (gbm). A deformation model based on an equilibrium between work-hardening and recovery processes has been developed. The decrease in dislocation density is due to both gbm associated with grain growth and the formation of boundaries by recrystallization. The value of the dislocation density along the Byrd and GRIP ice cores obtained thanks to the model is about 1×10^{11} m^{-2}. This value is in agreement with data from synchrotron X-ray diffraction on samples taken along the core. This deformation model can account for the deformation of polar ice at low stresses. It is shown that the flow law with a stress exponent lower than 2 can be related to the efficiency of gbm as a recovery process.

Keywords: polar caps; ice sheets; creep; deformation; fabric; mechanical properties

1. Introduction

Knowledge of the rheological properties of ice is of great interest for the modelling of the flow of polar ice sheets and also for the understanding of the mechanical behavior of materials deformed at high temperature. It is difficult to obtain quantitative measurements of the mechanical behavior of ice in the laboratory under low stress conditions typical of that found in polar ice sheets. Mechanical tests at stresses lower than 0.1 MPa take a long time (tens to hundreds of years) to reach large strains and mathematical extrapolation of results from higher stresses introduces significant uncertainty. As a consequence, conflicting results have been published. However, much progress has been made with the study of ice structure in deep ice cores recently retrieved from Antarctica and Greenland and with borehole deformation measurements.

At high stresses the rheology of ice is governed by intracrystalline dislocation slip mainly on basal planes [1]. The stress exponent for steady state flow is 3 over the stress range 0.1–0.8 MPa [1–3].

At low stresses, a stress exponent lower than

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3 is suggested by some borehole deformation measurements [4] and bubbly ice densification [5]. Some laboratory tests also suggest a stress exponent lower than 2 [6–9]. Such behavior was also obtained by Goldsby and Kohlstedt [10,11] from laboratory tests on very fine grained ice. Goldsby and Kohlstedt considered grain boundary sliding as the dominant deformation mechanism in polar ice sheets. In disagreement with this conclusion, Duval et al. [12] showed that the kinetics of development of lattice preferred orientations (fabrics) in polar ice sheets is incompatible with grain boundary sliding dominated creep as this is not a fabric building deformation process.

The aim of this work is to show that the deformation of polar ice at low stresses can be produced by intracrystalline slip accommodated by grain boundary migration (gbm) associated with normal grain growth or rotation recrystallization. Such a mechanism was suggested by Pimienta and Duval [13] and Alley [14] for ice and for high stacking fault energy metals by Gourdet et al. [15,16]. Using data from three deep ice cores from Antarctica and Greenland, we have developed a deformation model based on work-hardening and recovery processes. This deformation model does account for the deformation of ice in the major part of polar ice sheets.

2. Grain growth and rotation recrystallization in ice sheets

The evolution of the ice structure in polar ice sheets is achieved via normal grain growth, rotation recrystallization and migration recrystallization [14,17,18]. In the upper layers of ice sheets (several hundreds of meters), the mean grain size increases with depth. A parabolic growth relationship between grain size and time was found for several sites in Antarctica and Greenland [19]. The driving force for this *normal grain growth* is related to the decrease in free energy that accompanies a reduction in grain boundary area. Compared with metals, the driving force $3\gamma_{gb}/D$ is low, i.e. less than 100 J m^{-3}, since the grain size D is generally larger than 1 mm and the grain boundary free energy $\gamma_{gb} = 0.065$ J m^{-2} [14].

Below this normal grain growth region, heterogeneous deformation within grains leads to the formation of sub-boundaries by the gathering of dislocations. The misorientation of sub-boundaries increases as deformation proceeds and high angle boundaries develop, leading to the creation of new grains. This mechanism is termed *rotation recrystallization* by the geological community but is also referred to as *continuous recrystallization*. For the Byrd ice core, Alley et al. [22] indicate that polygonization processes associated with rotation recrystallization counteract further normal grain growth below 400 m depth. The same explanation was given by Castelnau et al. [23] for the halt in grain growth below 650 m in the GRIP ice core. At Vostok (East Antarctica), due to the continuous increase in temperature from the surface, the transition from grain growth to rotation recrystallization is difficult to determine from the grain size profile [18]. From Lipenkov (personal communication), the first sub-boundaries are observed under polarized light below 700 m, and normal grain growth is therefore thought to cease at about this depth.

Under rotation recrystallization conditions grain boundaries migrate in the same low velocity regime as that associated with normal grain growth, a regime which is characteristic of impurity loaded boundaries [24]. The driving force for gbm is both the strain energy and the surface energy associated with grain boundary curvature.

In the deepest hundreds of meters of the central parts of ice sheets, the temperature can be higher than −10°C, reaching the melting point at the interface between ice and rock. In this zone, rapid migration of grain boundaries can occur between dislocation-free nuclei and deformed grains [24]. This recrystallization regime is generally referred to as *migration recrystallization* by geologists and *discontinuous recrystallization* in materials science. The high gbm rate is attributed to a drastic change in the grain boundary mobility at a critical temperature close to −10°C [8]. However, a critical driving force is also needed to initiate this recrystallization process. A power flow law with a stress exponent equal to 3 is associated with this recrystallization regime [14].

3. Deformation model for ice at low stresses

We suggest that the deformation of polar ice at low stresses, when migration recrystallization does not occur, is produced by intracrystalline slip accommodated by gbm associated with normal grain growth or rotation recrystallization.

The following processes are taken into account: the slip of dislocations, the formation of sub-boundaries and boundaries and the annihilation of dislocations by moving boundaries. The change in dislocation density dρ during deformation is considered to be due to work-hardening and recovery as:

$$\mathrm{d}\rho = \left(\frac{\delta\rho}{\delta\varepsilon}\right)_t \mathrm{d}\varepsilon + \left(\frac{\delta\rho}{\delta t}\right)_\varepsilon \mathrm{d}t \quad (1)$$

The increase of the dislocation density $(\delta\rho/\delta\varepsilon)_t$ is mainly due to dislocation accumulation along basal planes. Recovery is supposed to come from gbm associated with grain growth in the upper part of ice sheets and from both the formation of grain boundaries and gbm when rotation recrystallization occurs.

The increase in dislocation density by work-hardening can be deduced from the Orowan relationship:

$$\frac{\mathrm{d}\rho^+}{\mathrm{d}t} = \frac{\dot{\varepsilon}}{bD} \quad (2)$$

where $\dot{\varepsilon}$ is the strain rate and b the Burgers vector; the dislocation free path D of the mobile dislocations is assumed to correspond to the grain size. In the calculations, dislocations in subgrain boundaries are considered as free dislocations and subgrain boundaries are not considered as obstacles to dislocation glide. The decrease in the total dislocation density is due to gbm during normal grain growth but to both gbm and the formation of grain boundaries during rotation recrystallization. The effect of dynamic recovery by the annihilation of dislocations by climb is not considered.

As a boundary moves, dislocations located in the volume swept by the mobile grain boundary disappear. The reduction of the total dislocation density by gbm is given by:

$$\frac{\mathrm{d}\rho_1^-}{\mathrm{d}t} = \frac{\alpha\rho K}{D^2} \quad (3)$$

where K is the grain boundary migration rate. α is a coefficient exceeding 1, which makes it possible to take into account a higher dislocation density near grain boundaries.

When only normal grain growth occurs, the variation with time of the grain size is given by [19]:

$$D^2 = D_0^2 + Kt \quad (4)$$

The evolution of the dislocation density with time can be deduced from Eqs. 2–4 as follows:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{\dot{\varepsilon}}{bD} - \alpha\frac{\rho K}{D^2} \quad (5)$$

For a given strain rate $\dot{\varepsilon}$, this equation leads to:

$$\rho(t) = \frac{2\dot{\varepsilon}}{3Kb}\left(D - \frac{B}{D^2}\right) \quad (6)$$

where D varies with time following Eq. 4, and B is a constant depending on the initial value of the dislocation density.

As the derivative of this equation is always positive, the dislocation density is continuously increasing in the normal grain growth region. This is coherent with the fact that grain growth is not affected by deformation.

An equilibrium can be reached when rotation recrystallization begins to occur.

The decrease of the dislocation density by the formation of grain boundaries is calculated assuming that subgrains misoriented by a mean angle θ contain only geometrically necessary dislocations.

The dislocation density associated with subgrains of a mean size D is therefore given by:

$$\rho_{\mathrm{sgb}} = \frac{2\theta}{bD} \quad (7)$$

We assume that sub-boundaries transform into grain boundaries when θ is equal to θ_c.

The total area of grain boundaries per unit volume S_{gb} corresponding to an equidimensional network of cubic grains with a mean grain size D is given by:

$$S_{gb} = \frac{3}{D} \quad (8)$$

From Eqs. 7 and 8 and with $\theta = \theta_c$, the theoretical dislocation density corresponding to S_{gb} is then expressed as:

$$\rho_{gb} = \left(\frac{2\theta_c}{3b}\right) S_{gb} \quad (9)$$

Thus, the reduction in the dislocation density by the formation of new grain boundaries is related to dS^+_{gb} by:

$$\frac{d\rho^-_2}{dt} = \left(\frac{2\theta_c}{3b}\right)\frac{dS^+_{gb}}{dt} \quad (10)$$

The variation of the dislocation density with time can be obtained from Eqs. 2, 3 and 10 if the variation of S_{gb} is known.

From Eq. 8, the increase in grain size associated with grain growth leads to a decrease in the total area of grain boundaries per unit volume corresponding to:

$$\frac{dS^-_{gb}}{dt} = \left(\frac{3}{D^2}\right)\frac{dD}{dt} \quad (11)$$

For a steady state, $dS_{gb} = 0$ and grain size does not change with time. This behavior is observed in ice sheets when rotation recrystallization occurs and if temperature does not change with depth [20,21]. As a consequence $dS^-_{gb} = dS^+_{gb}$ and:

$$\frac{d\rho^-_2}{dt} = \left(\frac{2\theta_c}{bD^2}\right)\frac{dD}{dt} \quad (12)$$

Under such conditions, a steady state flow law can be reached if $d\rho^-/dt = d\rho^+/dt$. From Eqs. 2, 3 and 12, the strain rate is then related to the equilibrium dislocation density ρ by:

$$\dot{\varepsilon} = \frac{bK}{D}\left(\alpha\rho + \frac{\theta_c}{bD}\right) \quad (13)$$

The gbm rate K is assumed to be equal to $2D(dD/dt)$, i.e. the driving force for gbm is the same as for normal grain growth at the transition between normal grain growth and rotation recrystallization.

From Eq. 13, knowing $\dot{\varepsilon}$, the value of D at the transition and an estimation of θ_c we can evaluate the equilibrium value of ρ.

4. Application to ice sheets

The variation of the dislocation density with depth was calculated along two deep ice cores: the Byrd ice core (West Antarctica) and the GRIP ice core (Greenland). The main characteristics of these ice cores are given in Table 1. Calculations were limited to the first 1000 m for the Byrd core and 1500 m for the GRIP core where temperature does not significantly change with depth. The behavior along the Vostok core (Antarctica) is also considered.

Due to the very low surface slope at these locations, horizontal shear is neglected. We also assume that the vertical strain rate is uniform along the studied length of the cores. As a consequence $\dot{\varepsilon}$ is constant along the cores. Rotation recrystallization was considered to occur below 400 m at Byrd [22] and below 650 m at GRIP [18].

Calculations were done with $\alpha = 1$, $\theta_c = 5°$ and

Table 1
Main characteristics of borehole sites studied

Site	Surface temperature (°C)	Length of the core (m)	Vertical strain rate (s^{-1})	K ($m^2\ yr^{-1}$)	D_{eq} (mm)
Byrd	−28	2164	2.5×10^{-12}	1.1×10^{-8}	6.3
GRIP	−32	3030	2.5×10^{-12}	3.8×10^{-9}	4
Vostok	−56.5	3625	2.3×10^{-13}		

D_{eq} is the equilibrium grain size and K the gbm rate.

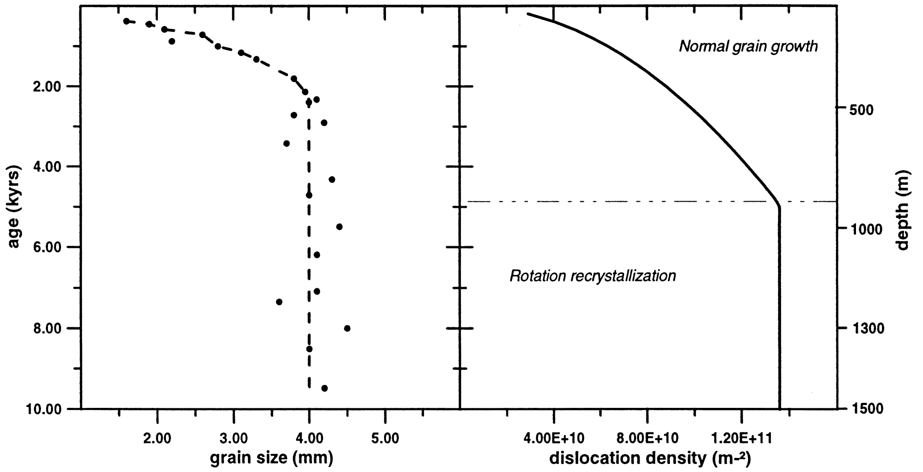

Fig. 1. Left: observed crystal size as a function of age and depth in the GRIP ice core (Greenland) [21]. Right: calculated dislocation density as a function of age and depth.

with $\rho = 1 \times 10^{10}$ m^{-2} at time $t = 0$. This value of the dislocation density corresponds to a realistic value for annealed ice [12]. The reduction of the dislocation density by gbm is therefore obtained by assuming a homogeneous dislocation distribution within grains ($\alpha = 1$). The value of θ_c was based on visual observations of sub-boundaries in polar ice. Figs. 1 and 2 show the variation with depth (and time) of the dislocation density obtained with the model described above. The measured variation of the crystal size along the two cores is also given. The dislocation density increases with depth according to Eqs. 2, 3 and 12 down to a critical depth and is constant below that depth. The steady state dislocation density found at GRIP, of about 1.2×10^{11} m^{-2} is equivalent to a stored energy of 36 J m^{-3}. This value is of the same order as the driving force for grain

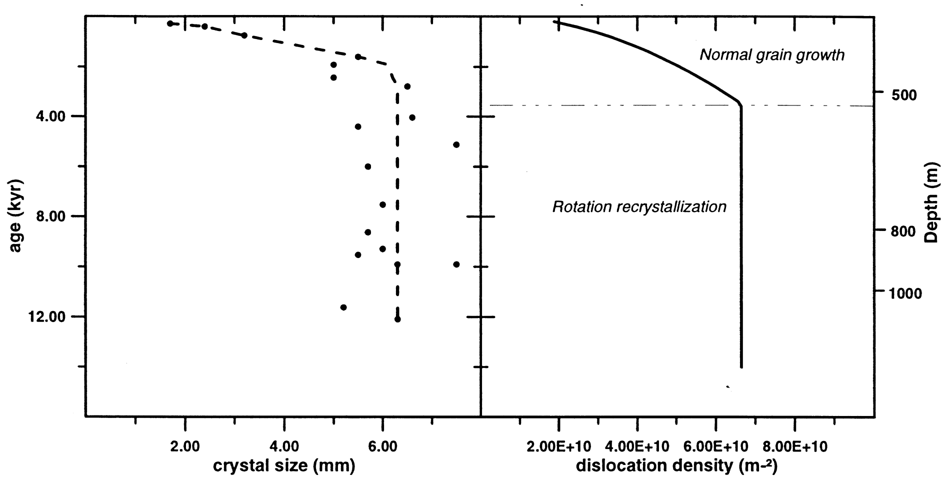

Fig. 2. Left: observed crystal size as a function of age and depth in the Byrd ice core (Antarctica) [20]. Right: calculated dislocation density as a function of age and depth.

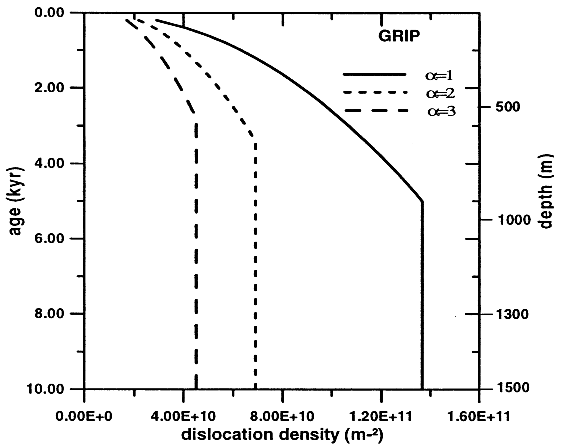

Fig. 3. Variation of the evolution of dislocation density as a function of age and depth, with the variation of the parameter α, along the GRIP ice core (Antarctica).

growth ($3\gamma_{gb}/D$), which is in agreement with the assumption of equilibrium between grain growth and rotation recrystallization at the transition depth. A similar result is found for the Byrd ice core with a slightly lower value for the dislocation density.

As shown in Fig. 3, the efficiency of gbm as a recovery process could be even higher if dislocations are preferentially located near boundaries. With $\alpha=3$ the dislocation density at equilibrium for the GRIP core would be lower than 6×10^{10} m^{-2}. The corresponding energy would therefore be lower than the driving force for normal grain growth which is not in agreement with the occurrence of rotation recrystallization. With α between 2 and 3, the equilibrium is reached at a depth closer to the observed transition between grain growth and rotation recrystallization.

A clear transition between grain growth and rotation recrystallization as displayed in the Byrd and GRIP ice cores can not be observed in the Vostok core since temperature increases almost from the surface. For this site, the dislocation density calculated with a similar model by neglecting the reduction of the dislocation density by the formation of grain boundaries reaches a maximum value at 1000 m depth, of about 1.3×10^{11} m^{-2} [18]. This value is therefore an upper bound since rotation recrystallization is observed below 700 m.

5. Discussion

The above results show that the deformation model based on the absorption of dislocations by gbm and by the formation of grain boundaries by rotation recrystallization can account for the deformation of polar ice. For the three sites studied, the dislocation density at equilibrium is found to be near 1×10^{11} m^{-2}. On the other hand, a preliminary study of the microstructure of ice crystals of the GRIP and Vostok cores by synchrotron X-ray topography indicates that the dislocation density is also of the same order as that deduced from this model [12]. The comparison between our calculations and those done by de La Chapelle et al. [18], who used a similar model but without taking into account recrystallization shows that the variation of the dislocation density with depth is significantly altered by the formation of boundaries. Steady state cannot be reached with the model used by de La Chapelle et al., and the dislocation density reaches a maximum value of about 2×10^{11} m^{-2} along the GRIP core.

Calculations of the dislocation density were done assuming that it is not dependent on the orientation of grains. Only average values for the polycrystal were therefore considered. The high plastic anisotropy of the ice crystal causes the development of a non-uniform stress field in the polycrystal [1]. The self-consistent theory, which gives the stresses and strain rates in each grain, predicts behavior with a minimum stress and a maximum strain rate for grains well oriented for basal slip [25]. Therefore, rotation recrystallization could be initiated in grains with high stored energy even though the average driving force for normal grain growth is higher than that for recrystallization [18]. The development of this internal stress field is considered to be associated with primary creep. As a consequence, the distribution of internal stress at the end of primary creep should have amplitude peaks at grain boundaries so that the value of the parameter α should be higher than 1 as suggested above. The accommodation of slip by gbm and recrystallization should attenuate the internal stress field caused by the strong plastic anisotropy of ice.

Concerning the flow law and especially the relation between strain rate and stress, only qualitative conclusions can be drawn. Indeed, Eq. 13 obtained for steady state does not allow us to determine the flow law. The crystal size D and the dislocation density must depend on stress. However, it would seem hazardous to go further with the physical model described above. The validity of the classical equation $\rho = \beta(\sigma/Gb)^2$ deduced from a 3D network of dislocations is questionable for ice since basal slip contributes almost totally to the deformation of grains. Resistance for the glide of dislocations on basal planes should occur through accumulation of dislocations against grain boundaries.

By absorbing dislocations located in the volume swept, mobile grain boundaries prevent the formation of internal stresses. The efficiency of this recovery process should increase as strain rate decreases. As a consequence, the dependence of ρ on stress is reduced and the value of 2 for the exponent relying ρ to σ represents an upper bound. A stress exponent between 1 and 3 is therefore expected. The decrease of the stress exponent for stresses lower than 0.1 MPa seems compatible with the occurrence of gbm and rotation recrystallization.

The suggestion of Goldsby and Kohlstedt [11] that the deformation of ice at stresses lower than 0.1 MPa with $n = 1.8$ is dominated by grain boundary sliding is not in accordance with the model described above. Grain boundary sliding could be invoked as a deformation mode like non-basal slip, but its contribution to the total strain will stay very low as long as basal slip is a soft deformation mode. As mentioned above, the development of fabrics in polar ice sheets is not in agreement with grain boundary sliding as a dominant deformation mechanism. A good simulation of the fabric development in polar ice sheets was obtained with several polycrystal deformation models based on intracrystalline slip [24,26].

6. Conclusion

Data on the structure of deep ice cores support the assumption of the preponderance of dislocation slip for the deformation of polar ice. Intracrystalline slip appears to be accommodated by gbm associated with normal grain growth or rotation recrystallization.

A physical deformation model based on equilibrium between work-hardening and recovery has been developed. Gbm appears to be an efficient process for the accommodation of slip. The dislocation density along the first 1500 m of the Byrd and GRIP ice cores is found to continuously increase in the normal grain growth region and to keep a constant value when rotation recrystallization occurs. Calculations were done considering mean values for the polycrystal. Indeed, variations in the dislocation density with the orientation of grains are expected due to the high plastic anisotropy of the ice crystal. The low value of the stress exponent for the flow law ($n < 3$) appears to be related to the efficiency of gbm in relieving the internal stress field.

Improvements concerning the variation of the stored energy with the orientation of grains are expected thanks to X-ray diffraction experiments.***[FA]***

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