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BUCKLING AND POST-BUCKLING OF LONG PRESSURIZED ELASTIC THIN-WALLED TUBES UNDER IN-PLANE BENDING

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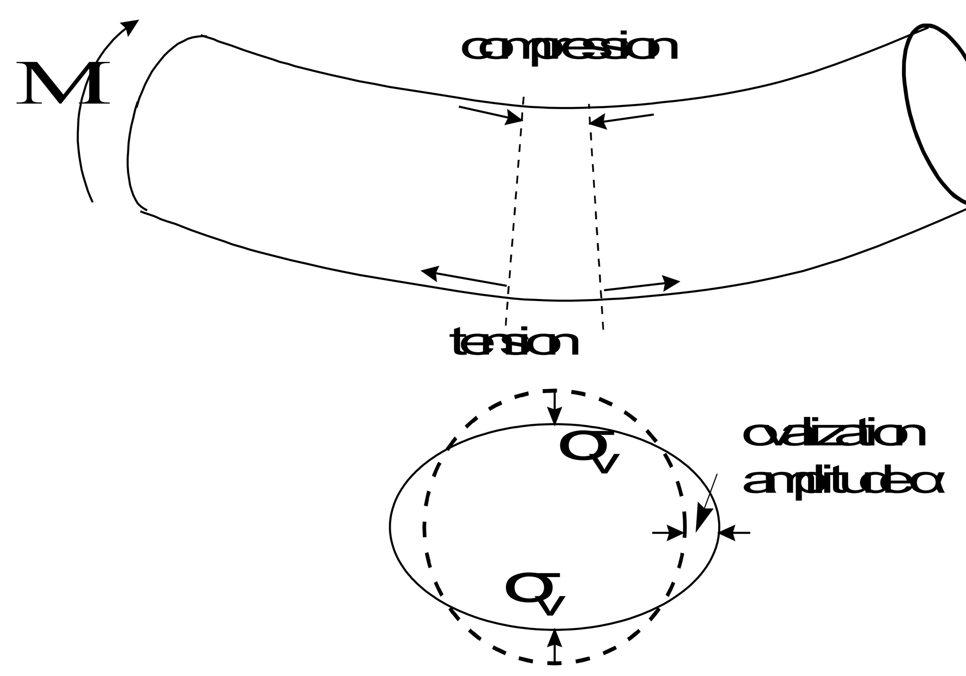
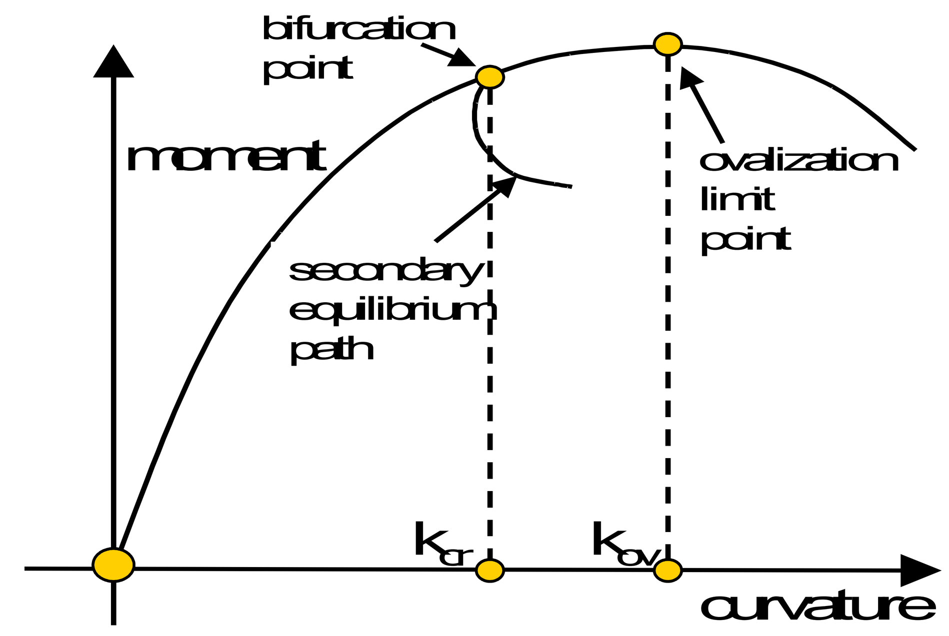
ABSTRACT

The present project focuses on the structural stability of long uniformly pressurized thin elastic tubular shells subjected to in-plane bending, using a special-purpose nonlinear finite element technique. In particular, bifurcation on the prebuckling ovalization equilibrium path is detected, and the post-buckling path is traced. Furthermore, the influence of pressure (internal and/or external) as well as the effects of radius-to-thickness ratio, initial curvature and initial ovality on the bifurcation moment, curvature and the corresponding wavelength, are examined. The local character of buckling in the circumferential direction is also demonstrated, especially for thin-walled tubes. This observation motivates the development of a simplified analytical formulation for tube bifurcation, which considers the presence of pressure, initial curvature and ovality, and results in closed-form expressions of very good accuracy, for tubes with relatively small initial curvature.

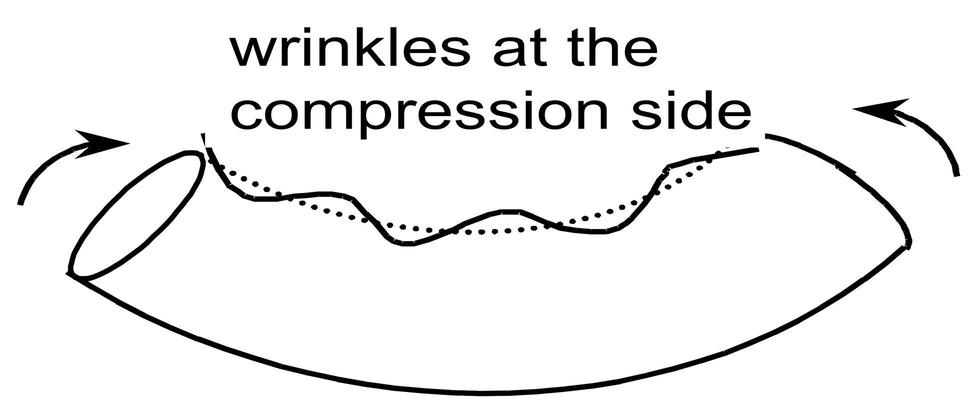
GLOSSARY

Ovalization Instability: The main characteristic of tube response under bending is the distortion (ovalization) of its cross-section due to the inward stress components σv. The ovalization mechanism results in loss of stiffness in the form of limit point instability, referred to as 'ovalization instability' or Brazier effect. In this section, the nonlinear bending response is examined, in terms of ovalization instability.

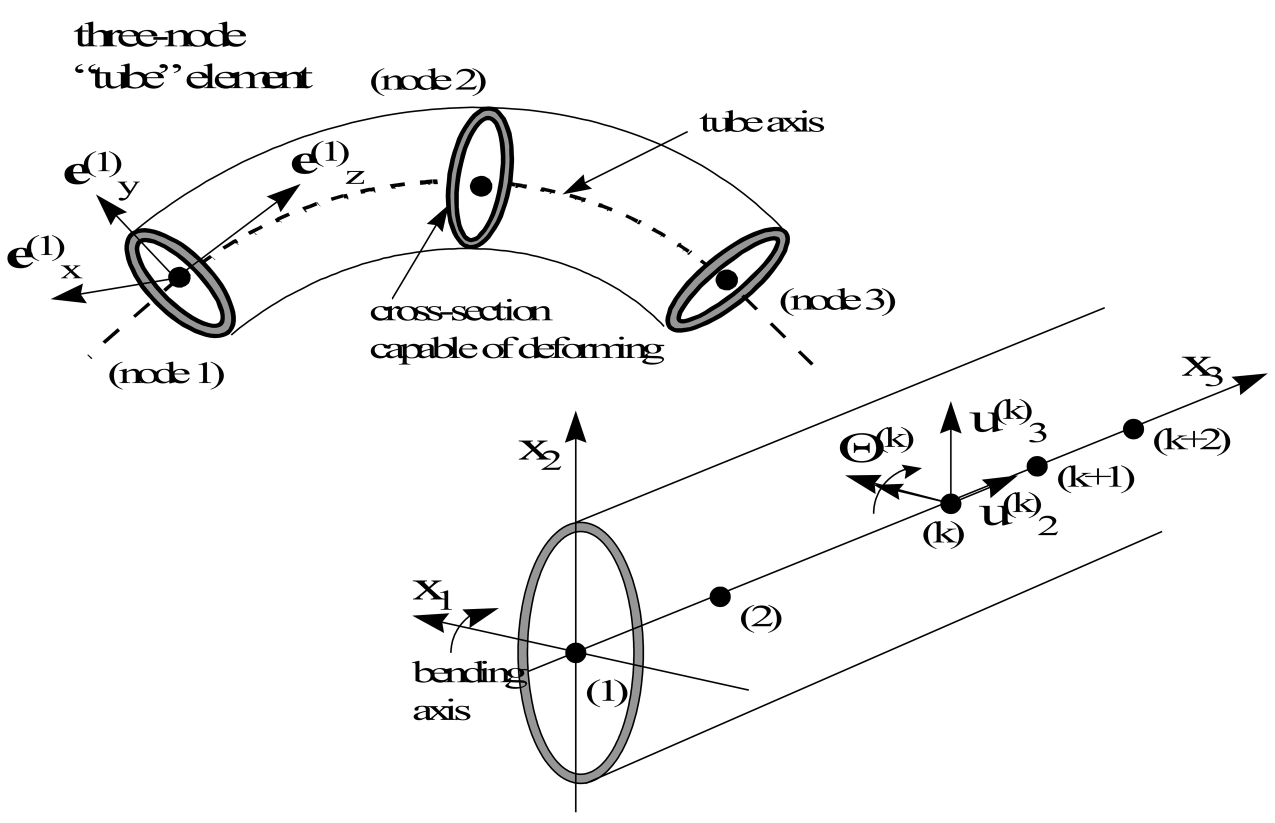




Bifurcation Instability: The increased axial stress at the compression side may cause bifurcation instability (buckling) of the ovalized cross-section in the form of longitudinal wavy-type 'wrinkles'. Previous works have indicated that bifurcation occurs usually before an ovalization limit moment is reached.



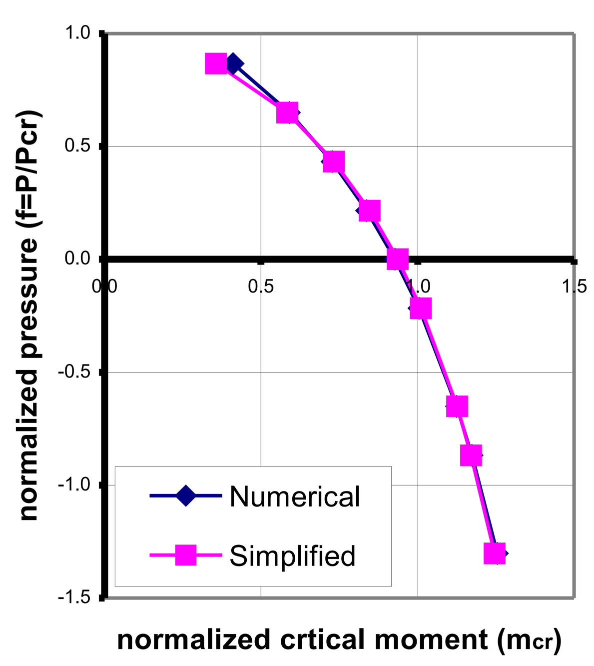
NUMERICAL TECHNIQUE

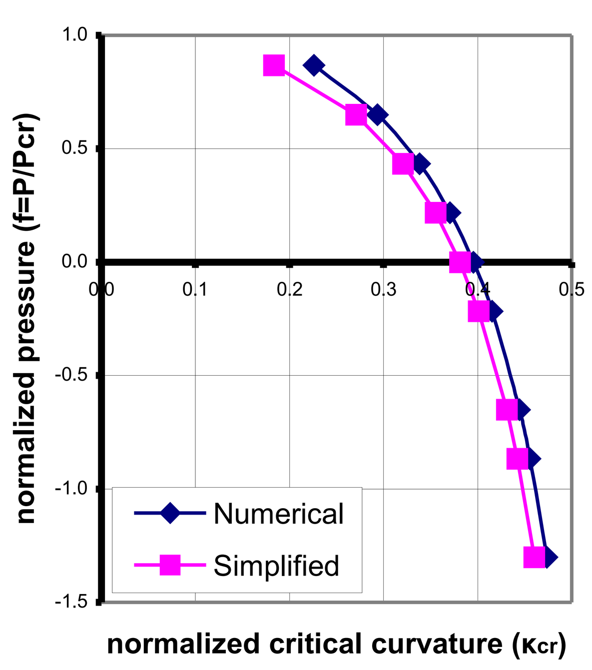


buckling at $\sigma_{L,cr} = \dfrac{E}{\sqrt{3(1-\mu^2)}}\dfrac{t}{r_\theta^0}$

$$\frac{1}{r_\theta^0} = \frac{(1-f)-3\kappa_{cr}^2-3\kappa_{cr}\,\kappa_{in}}{r(1-f)} \quad \text{and} \quad \sigma_{L,cr} = \frac{E}{\sqrt{1-\mu^2}}\frac{t}{r}\kappa_{cr}[1-\frac{(\kappa_{cr}+\kappa_{in})^2}{1-f}]$$

$$\Rightarrow\; -\frac{1}{\sqrt{3}}\left(1-\frac{3\kappa_{cr}(\kappa_{cr}+\kappa_{in})}{1-f}\right)+\kappa_{cr}\left(1-\frac{(\kappa_{cr}+\kappa_{in})^2}{1-f}\right)=0$$





ANALYTICAL EXPRESSIONS

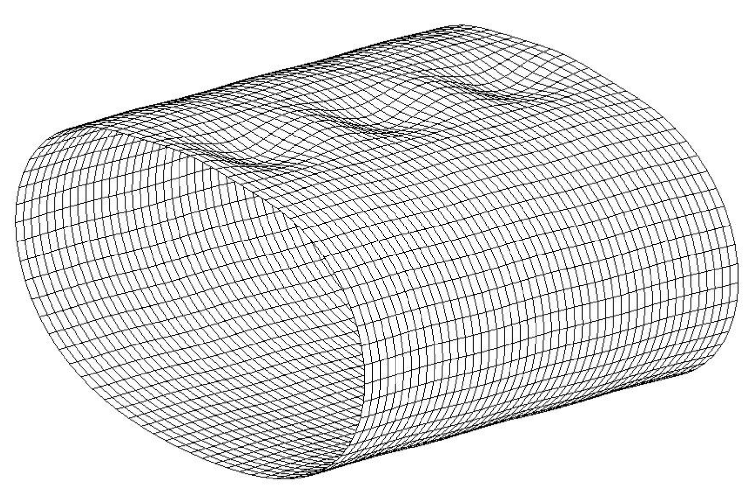
$$\Pi = U_L + U_C - W_P - M\,k \quad \textit{potential energy (pressurized bending)}$$

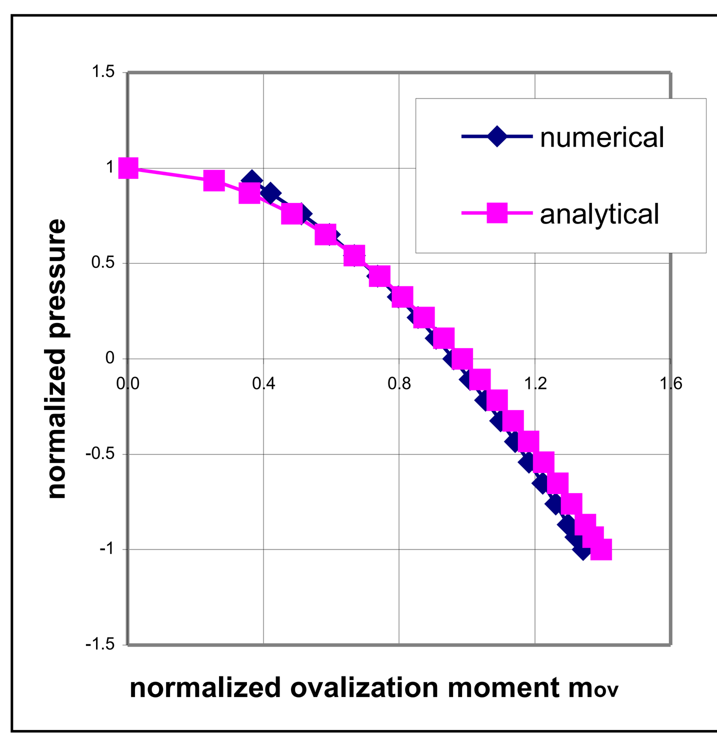
Equilibrium

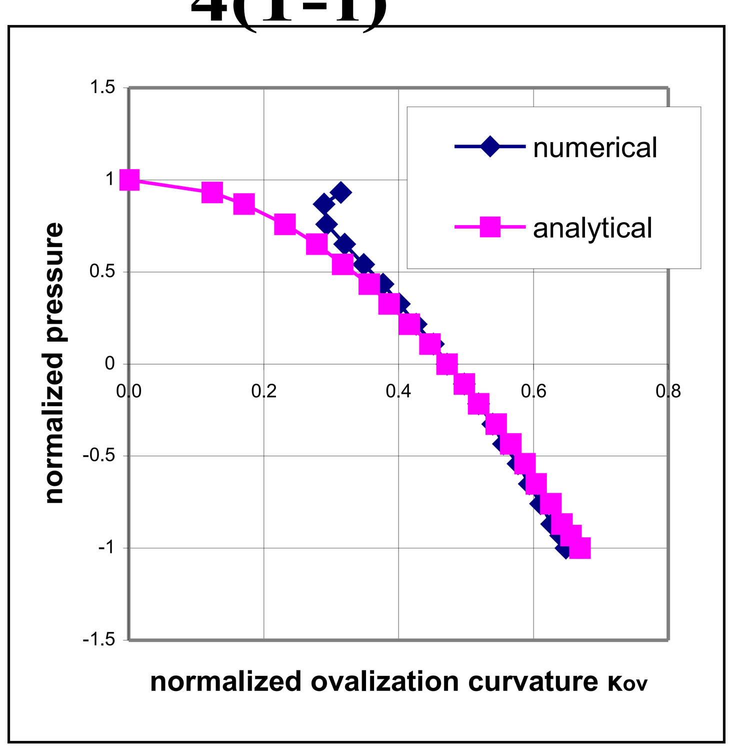
$$\frac{\partial \Pi}{\partial \alpha}=0 \;\Rightarrow\; \alpha = \frac{\kappa r(\kappa+\kappa_{in})}{1-f}$$

$$\frac{\partial \Pi}{\partial k}=0 \;\Rightarrow\; m = \kappa\pi\left(1-\frac{3(\kappa+\kappa_{in})(2\kappa+\kappa_{in})}{4(1-f)}\right)$$

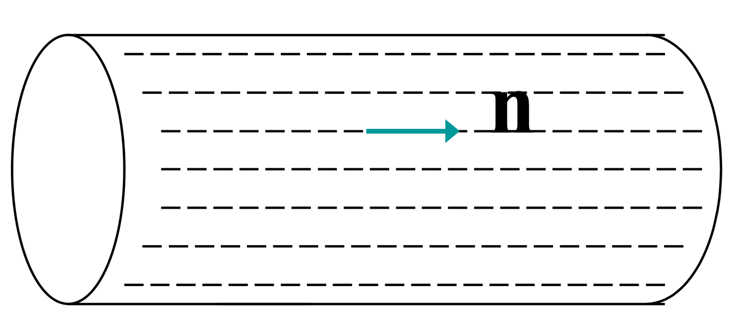
(normalized relations)





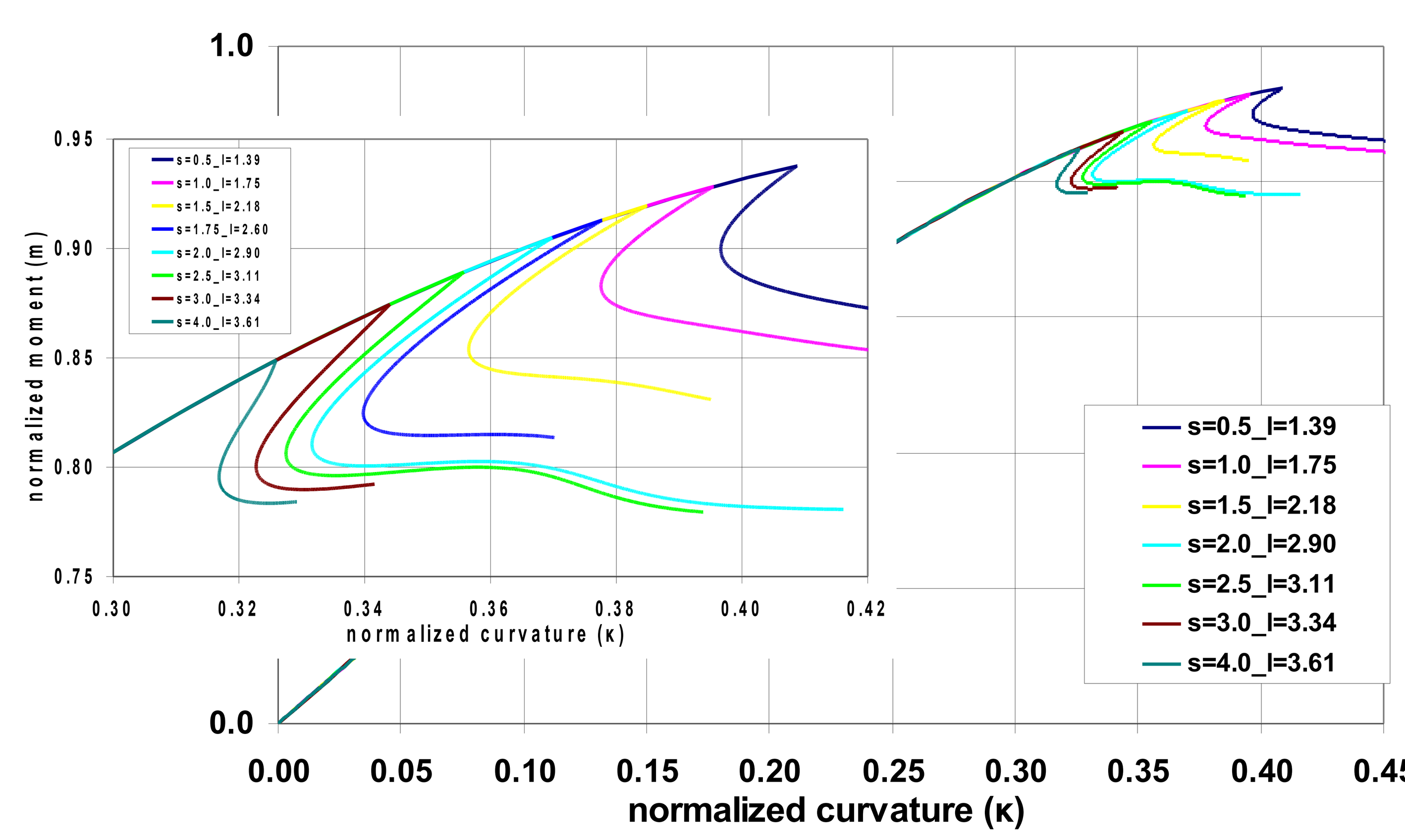


FUTURE WORK



Transversely isotropic elasticity

s: anisotropy parameter



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